# The Total Factor Productivity Index as a Ratio of Price Indexes.

## *L'indice della produttività totale dei fattori come rapporto di indici di prezzo.*

Lisa Crosato and Biancamaria Zavanella

**Abstract** Ever since the seminal work by Robert Solow (1957), total factor productivity (TFP hereafter) has been associated with the movements in the production function caused by technical progress. This has largely influenced the choice of the index numbers involved in the productivity measurement. In fact, the methodology currently in use worldwide is based on the superlative Tornqvist index (Diewert, 1976). In this paper we observe the empirical evidence about TFP in a few countries and propose to re-define the TFP as the ratio of the price index of input to the price index of outputs, using the Sato-Vartia formula.

**Abstract** *La produttività totale dei fattori viene associata con i movimenti della funzione di produzione causati dal progresso tecnico a partire dal lavoro fondamentale di Robert Solow (1957). Questo ha ampiamente influenzato il modo di misurare la produttività e in particolare la scelta dei numeri indice coinvolti, tanto che la metodologia universalmente usata al momento è basata sull'indice superlativo di Tornqvist (Diewert, 1976). In questo lavoro partiamo dell'evidenza empirica sull'andamento della produttività totale dei fattori in alcuni paesi e ridefiniamo l'indice TFP come rapporto di indici di prezzo, usando la formula di Sato-Vartia.*

**Key words:** productivity, index numbers, Sato-Vartia, Tornqvist

## 1 Introduction

Productivity measures the extent to which an economic system transforms the available resources in goods and services. More specifically, we can define productivity as the ratio between the volume of output and of the input utilized to generate it or between the results achieved by the economic system of a region/country and the imputed factors (OECD, 2001). On the empirical side, for any production unit, the

Università di Milano-Bicocca - DEMS - Via Bicocca degli Arcimboldi, 8, 20126 Milano.
e-mail: lisa.crosato@unimib.it; biancamaria.zavanella@unimib.it

total factor productivity index is defined as an output quantity index divided by an input quantity index (Balk, 2010).

The interest in productivity measurement has been strong ever since the late thirties, starting from the introductory papers by Evans and Siegel (1942) and Magdoff (1939). Later on, the seminal work of Solow (1957), briefly followed by the quarrel with Pasinetti (1959) intertwined the total factor productivity (TFP) with technical progress.

The current methodology in use at official statistics offices for measuring Total Factor Productivity (TFP hereafter) derives from the Solow model rooted into a neoclassical framework and ends up in the ratio of two quantity indexes: the Tornqvist quantity input index and an implicit output quantity index derived through the output price index of Paasche (OECD, 2001). This way of constructing the TFP index, however, is not consistent with the axiomatic theory of index numbers and leads to puzzling empirical patterns.

This paper proposes to rethink the TFP index in terms of the ratio of the input price index to the output price index, by means of the Sato-Vartia formula. The underlying idea builds on the empirical evidence on the TFP changes, which for several countries show a fluctuating behaviour hardly consistent with the mechanisms driving technological progress. On the contrary, the relative changes between input and output prices are of a more volatile nature so that our representation could be more consistent with the observed pattern of TFP. Furthermore, the use of Sato-Vartia formulas is in line with the index numbers theory since they satisfy a few axiomatic tests failed by the Tornqvist and the implicit quantity indexes.

Among other factors besides technological progress shaping the pattern of TFP and causing noise, we can surely include the relative input/output cost or price behaviour. So a look at the TFP index on the index price side could maybe give a more consistent interpretation of this puzzling behaviour.

The remaining of the paper is organized as follows: section 2 sketches the definition of TFP index by means of Sato-Vartia input and output price indexes, Section 3 illustrates the empirical evidence for Italy, section 4 concludes.

## 2 Methodology

Our starting point is the current methodology outlined in the OECD manual, which builds on the Cobb-Douglas production function and then factorizes the parameter representing the tecnhical progress or TFP. Under the usual neoclassical hypotheses, output elasticities with respect to capital and labour match the shares of added value going to the same factors.

Through standard mathematical tools the TFP index can be derived as an output index divided by the Divisia index synthetising the real variation of production factors, weighted by the corresponding shares (Star and Hall, 1976). However, as it is well known, the Divisia index cannot be applied to real data because the index is continuous and the data are discrete, and since Diewert (1976) it has been

approximated by the Tornqvist formula, chosen mainly on the basis of its relations to the possible functional forms for the production function. All this results in the definition of the TFP index as the ratio between an implicit quantity index for outputs (given by the nominal variation of value added over the Paasche price output index) and the Tornqvist index for input quantities. In formulas, let $n$ be the number of possible outputs, $L$ and $K$ the labour and capital inputs respectively, $V$ the value, $p_{ti}$ the output prices and $y_{ti}$ the output quantities for $i = 1,\dots,n$ , $w_t$ the cost and $L_t$ the quantity of labour, $u_t$ the cost and $K_t$ the quantity of capital. Then

$$u_t K_t + w_t L_t = \sum_{i=1}^{n} p_{it} y_{it}$$

indicates the aggregate value at time $t$ so that the value index is given by

$$\frac{u_t K_t + w_t L_t}{u_{t-1} K_{t-1} + w_{t-1} L_{t-1}} = \frac{\sum_{i=1}^{n} p_{it} y_{it}}{\sum_{i=1}^{n} p_{it-1} y_{it-1}} =_{t-1} V_t \tag{1}$$

Now the value index can be decomposed into the product of a price Paasche index and a Laspeyres quantity index for output ($_{t-1}P_t^{Po}$ and $_{t-1}Q_t^{Lo}$ respectively)

$$\frac{\sum_{i=1}^{n} p_{it} y_{it}}{\sum_{i=1}^{n} p_{it-1} y_{it-1}} =_{t-1} P_t^{Po} \cdot_{t-1} Q_t^{Lo} =_{t-1} V_t \tag{2}$$

According to the standard methodology of the OECD, the real variation of value added is then measured by the implicit quantity index

$$\sum_{i=1}^{n} p_{it} q_{it} /_{t-1} P_t^{Po} \tag{3}$$

and the TFP from $t-1$ to $t$ is defined by

$$_{t-1}V_t /_{t-1} P_t^{Po} =_{t-1} TFP_t \cdot_{t-1} Q_t^{TI} \tag{4}$$

with $_{t-1}Q_t^{TI}$ being the Tornqvist input quantity index.

However, the Tornqvist index does not satisfy the product test of index numbers and, as Balk (2010) points out, "the implicit quantity index does not satisfy the Identity Test". Therefore, the standard TFP index currently used results non consistent into the framework of axiomatic index number theory.

We propose to resort to the Sato-Vartia index instead, substituting it to the Tornqvist formula in (4)

$$_{t-1}V_t /_{t-1} P_t^{Po} =_{t-1} TFP_t \cdot_{t-1} Q_t^{SVI} \tag{5}$$

Once again, (5) says that the TFP index is given by the ratio of the output quantity index to the input quantity index with the usual interpretation as a technical progress indicator and shitfting of the production function. Recalling the decomposition of the nominal variation of the value added through the Sato-Vartia index (see for instance Martini (2001))

$$\frac{\sum_{i=1}^{n} p_{it} y_{it}}{\sum_{i=1}^{n} p_{it-1} y_{it-1}} = {}_{t-1}P_t^{SVo} \, {}_{t-1}Q_t^{SVo} = {}_{t-1}V_t \tag{6}$$

we can express the TFP as

$$\begin{aligned} {}_{t-1}TFP_t &= {}_{t-1}Q_t^{Lo} / {}_{t-1}Q_t^{SVI} \\ &= \frac{{}_{t-1}V_t}{{}_{t-1}V_t} \frac{{}_{t-1}Q_t^{Lo}}{{}_{t-1}Q_t^{SVI}} \\ &= \frac{{}_{t-1}P_t^{SVI}}{{}_{t-1}P_t^{Po}} \end{aligned} \tag{7}$$

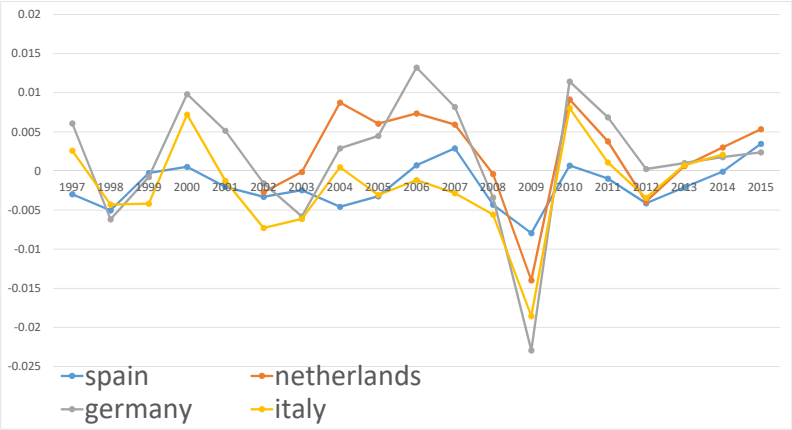
so that the TFP index can be represented, with no loss of information, as the ratio of the Sato-Vartia input cost index and the Paasche output price index. Furthermore, writing the TFP in this way we are explicitating that it is influenced by several different factors that maybe can contribute to justify the ups and downs of TFP detected empirically.

## 3 Empirical evidence

The empirical evidence we started working on in order to develop the idea underlying this work is based on data from the EUKLEMS database (www.euklems.net). Euklems collects several measures related to economic growth and productivity, at the industry level for all EU members from 1970 onwards and is the result of the homonymous project started in by the European Commission in order to provide quantitative input to evaluate competitiveness, economic growth and policy interventions.

Empirical evidence on log-changes in TFP over the last 18 years reveals a fluctuating behaviour that would indicate ups and downs in the technical progress hardly understandable (see Figure 1 for Italy, Germany, Spain and the Netherlands).

**Fig. 1** Log-ratio of ${}_{t-1}TFP_t$. Source: Our elaborations on EU-KLEMS data for the indicated countries.



## 4 Conclusions

Changes in empirical TFP do not always show the increasing pattern one would associate to the accumulation of technical progress, but for some countries seem rather fluctuating around a null average value. This is not consistent with the technical-progress-ratio-of-quantity-indexes equation exploited worldwide to measure TFP, but could be explained by other factors, for instance by the index used in the deflation process.

Taken the added value deflation technicalities as given and known (due to the widespread use of the chained quantity Laspeyres index), to gain a different point of view we have turned our attention to the input quantity indexes at the denominator, calculated through the Tornqvist index.

In an axiomatic framework, the Tornqvist index has been questioned (Balk, 2010) because it does not satisfy the factor reversal test neither the cofactor identity. In this paper we propose the use of the Sato-Vartia index, which encompasses both problems if the data necessary to construct it are available, as in the EU-KLEMS database.

Although the numerical result of the TFP index is the same with either Tornqvist or Sato-Vartia formula, the choice of the representation in terms of quantity or price indexes is not a pettifogging matter, because the construction of the index influences its intepretation and evaluation in terms of economic policy. For instance TFP indexes, both at the aggregate or sectoral levels are widely used as regressors in empirical economics but, as it comes to the data, shocks in productivity are, for some countries, all but evident. Our contribution provides a possible re-evaluation of this behaviour and suggests TFP index or its changes to be carefully used as explanatory variables.

## References

Balk, B. M. (2010). An assumption-free framework for measuring productivity change. *Review of Income and Wealth 56*(s1).

Diewert, W. E. (1976). Exact and superlative index numbers. *Journal of econometrics 4*(2), 115–145.

Evans, W. D. and I. H. Siegel (1942). The meaning of productivity indexes. *Journal of the American Statistical Association 37*(217), 103–111.

Magdoff, H. (1939). The purpose and method of measuring productivity. *Journal of the American Statistical Association 34*(206), 309–318.

Martini, M. (2001). *Numeri indice per il confronto nel tempo e nello spazio*. CUSL.

OECD (2001). Measurement of aggregate and industry level productivity growth. Technical report.

Pasinetti, L. L. (1959). On concepts and measures of changes in productivity. *The Review of Economics and Statistics*, 270–286.

Solow, R. M. (1957). Technical change and the aggregate production function. *The review of Economics and Statistics*, 312–320.

Star, S. and R. E. Hall (1976). An approximate divisia index of total factor productivity. *Econometrica: Journal of the Econometric Society*, 257–263.