A functional regression control chart for profile monitoring

Carta di controllo mediante regressione funzionale per il monitoraggio di profili

Fabio Centofanti, Antonio Lepore, Alessandra Menafoglio, Biagio Palumbo and Simone Vantini

Abstract In many applications, profile monitoring techniques are needed when the quality characteristic under control can be modeled as a function. Moreover, measures of other functional covariates are often available together with the functional quality characteristic. To combine the information coming from all the measures attainable, a new functional control chart is proposed for profile monitoring. It relies on the residuals of a function-on-function linear regression of the quality characteristic on the functional covariates. The effectiveness of the proposed monitoring scheme is illustrated on a real-case study about the monitoring of CO_2 emissions from a Ro-Pax ship owned by the shipping company *Grimaldi Group*.
Abstract *In molte applicazioni è necessario utilizzare metodi di controllo statistico di processo a caratteristiche di qualità modellabili come funzioni. Inoltre, è spesso possibile integrare nello schema di monitoraggio anche la disponibilità di osservazioni di covariate funzionali ad esse correlate. In questo lavoro, viene presentata una carta di controllo funzionale basata sui residui di un modello di regressione lineare funzionale della caratteristica di qualità sulle covariate ad essa associate. Le potenzialità dell'approccio proposto, vengono illustrate mediante un caso studio sul monitoraggio delle emissioni di CO_2 di una nave da carico e passeggeri.*

Key words: statistical process control, profile monitoring, functional data analysis, functional linear regression

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1 Introduction

New statistical process control (SPC) methods have to be developed in order to handle complex data, whose collection and storage is nowadays facilitated by modern data acquisition technologies. In many practical situations the quality characteristic of the process can be modelled as a function defined on a compact domain. Data of such kind are the foundation of a rapidly expanding area of statistics referred to as *functional data analysis* (FDA [14, 4]). SPC methods that allow monitoring and controlling such processes are better known as *profile monitoring* techniques [12].

Measures of functional covariates are often available together with the functional quality characteristic. In this communication we build a novel control charts scheme [11] which fully exploit the additional information content of those functional covariates to improve the profile monitoring of the quality characteristic. The proposed chart is referred to as *functional regression control chart* (FRCC) and provides an extension of the *regression* (or *cause-selecting*) *control chart*, which arises in the multivariate context [10, 7, 8, 16, 15]. The effectiveness of the proposed control chart is demonstrated by means of a real-case study dealing with monitoring CO_2 emissions and identifying (negative) shifts after a specific energy efficiency initiative (EEI) that has been performed on a Ro-pax ship owned by the shipping company *Grimaldi Group*.

2 The proposed control chart

Let $Y(t)$ represent the functional quality characteristic (hereinafter referred to as *response variable* or simply *response*) and $\boldsymbol{X}(t) = (X_1(t), \ldots, X_p(t))^\top$ be the functional covariates (hereinafter referred to as *predictor variables* or simply *predictors*) with $t \in \mathscr{S}$, a compact set in $\mathbb{R}$, which are assumed to be elements of $L_2(\mathscr{S})$ (the Hilbert space of square integrable functions defined on the closed interval $\mathscr{S}$). In this setting, we model the predictors as influencing the response according to the following multivariate functional linear regression model

$$Y^Z(t) = \int_{\mathscr{S}} (\boldsymbol{\beta}(s,t))^\top \boldsymbol{X}^Z(s)\, ds + \varepsilon(t), \tag{1}$$

with $Y^Z(t)$ and $\boldsymbol{X}^Z(t) = \left(X_1^Z(t), \ldots, X_p^Z(t)\right)^\top$ the point-wise standardized response and predictor variables and $\boldsymbol{\beta}(s,t) = (\beta_1(s,t), \ldots, \beta_p(s,t))^\top$ the vector of the regression functional parameters. Given n iid observations $(Y_i(t), \boldsymbol{X}_i(t)),\ i = 1, \ldots, n$ (i.e., Phase I dataset), an estimator $\hat{\boldsymbol{\beta}}(s,t)$ of the coefficent vector $\boldsymbol{\beta}(s,t)$ is obtained by considering the truncated versions of the univariate and multivariate Karhunen-Loève's basis expansions [6] of $Y^Z(t)$ and $\boldsymbol{X}^Z(t)$, respectively. The rationale behind the FRCC is monitoring the functional residual

$$e(t) = Y^Z(t) - \hat{Y}^Z(t), \tag{2}$$

where

$$\hat{Y}^{Z}(t) = \int_{\mathscr{S}} \left(\hat{\boldsymbol{\beta}}(s,t)\right)^{\top} \boldsymbol{X}^{Z}(s)\, ds \tag{3}$$

is the prediction of $Y^{Z}(t)$ given $\boldsymbol{X}^{Z}(t)$ based on the linear model (1). For this purpose, we use the approach adopted in [5, 17, 13] where the residuals are decomposed via principal component analysis and the corresponding coefficients are monitored by means of the Hotelling's T^2 and the squared prediction error (SPE) control charts. However, other expansion techniques can be used as well. The control limits of the T^2 and SPE control charts are calculated as the percentiles of their empirical distributions obtained from the Phase I sample based on an overall Type I error α. This phase, along with the estimation of the sample version of Equation (1), will be referred to as Phase I.

For a new observation $(\boldsymbol{X}^{*}(t), Y^{*}(t))$, the residual and the associated T^2 and SPE statistics can be calculated. An alarm is issued if at least one of the latter two statistics violates the control limits (Phase II).

3 A real-case study

In this section an application to a real-case study of the FRCC is illustrated. In particular, real data are collected from a Ro-Pax ship owned by the Italian shipping company *Grimaldi Group* from December 2014 to October 2017. The *CO_2 emissions* per each voyage are considered as the response variable, whereas, the *sailing time*, the *speed over ground* and the *longitudinal* and *transverse wind components* are assumed as the predictors (further information on the variables can be found in [1, 3, 9]).

During February 2016 a specific EEI was performed that plausibly produced a negative shift in the response mean [3]. In light of this, observations collected before the considered EEI are used as the Phase I sample, whereas the remaining observations pertain to the Phase II. The overall Type I error α is set to 0.0027, as commonly done to set three-sigma limits on classical Shewhart control charts. To evaluate the FRCC performance, two competitor profile monitoring schemes proposed in [2] and [13], respectively, are considered as well. The first consists of monitoring the scores along the first principal components of the response by means of the Hotelling's T^2 and SPE control charts (hereinafter referred to as as RESP control chart). The second competitor is the index-based (INBA) control charting, which monitors the area under the response curve.

The performance of the considered control charts are evaluated by means of the *average run length* (ARL) [11] after the EEI, given that the in-control ARL is equal to $1/\alpha = 370$. The estimated ARLs, denoted with $\widehat{\text{ARL}}$s, are reported in Table 1. They clearly point out that the proposed control chart outperforms the competitors. Indeed, the $\widehat{\text{ARL}}$ achieved by the proposed FRCC is markedly lower than those of the RESP and INBA control charts.

Table 1 $\widehat{ARL}$s for the FRCC, RESP control chart and INBA control chart

	$\widehat{ARL}$
FRCC	18.60
RESP	34.09
INBA	50

4 Conclusion

A regression control chart is proposed to monitor a functional quality characteristic when observations of other functional covariates are available. It consists of monitoring the functional residuals coming from a function-on-function linear regression of the response, instead of the response. An application of the proposed control chart to a real-case study aiming to monitor the CO_2 emissions of a Ro-pax ship demonstrates that it outperforms two other popular alternatives proposed in the literature in identifying a shift on the response. However, further investigations should be done to asses the FRCC performance over different scenarios.

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