Changes in poverty concentration in U.S. urban areas

Cambiamenti nella concentrazione della povertà nelle città americane

Francesco Andreoli and Mauro Mussini

Abstract This paper explores the changes in urban poverty concentration in U.S. cities in the 1980-2016 period. Since poverty is unevenly distributed between neighborhoods in a city, poverty concentration is measured by calculating the Gini index of neighborhood poverty headcount ratios. The change in the index is broken down into components along different dimensions, notably time and space.

Abstract *L'articolo esamina i cambiamenti nella concentrazione della povertà nelle aree urbane americane dal 1980 al 2016. La concentrazione della povertà è misurata con l'indice di Gini, calcolato per l'incidenza della povertà a livello di quartiere. La variazione dell'indice è scomposta secondo la dimensione spaziale e quella temporale.*

Key words: poverty, spatial concentration, decomposition

1 Introduction

Inequalities in American cities can be observed since income and opportunities are unevenly distributed within the cities [4, 2]. In the same American metro area there may be neighborhoods where most of the residents are at the bottom of the income distribution in the city, and other neighborhoods whose residents are mostly at the top. When poor individuals are more likely to live in some neighborhoods, poverty tends to be concentrated in such neighborhoods that offer fewer economic opportunities for their residents, causing a reduction in economic mobility [3]. The most

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used measures of the degree of concentrated poverty across the neighborhoods in a city are based on the fraction of poor individuals who live in neighborhoods with high poverty levels. For example, according to the American Census definition, concentrated poverty can be measured as the fraction of poor individuals in the city who live in neighborhoods where at least 40% of residents are poor. We use a conventional inequality index to measure urban poverty concentration. This takes the form of the Gini index of neighborhood poverty headcount ratios. The more unequally distributed are poverty proportions across the city neighborhoods with respect to the citywide distribution, the larger is urban poverty concentration. This concentration measure captures a form of segregation of poor individuals across the city neighborhoods: when urban poverty concentration is high, there are neighborhoods with very high shares of local residents who are poor, and neighborhoods that are nearly poverty-free.

The Gini index of urban poverty is broken down into spatial components by using the Rey and Smith spatial decomposition of the Gini index [6]. In this way, we assess whether urban poverty is spatially concentrated within the city. We analyze the dynamics of concentrated poverty in American metro areas by considering the change in the Gini index of urban poverty from 1980 to 2016. First, building on Andreoli and Mussini [1], we break down the change over time in the Gini index of urban poverty into components that are attributable to different sources of the change in urban poverty concentration. Second, the change over time in each spatial component of the concentration index is decomposed.

2 Decomposing changes in concentrated poverty

We introduce some preliminary definitions and notation that are used to express the Gini index of urban poverty in a matrix form. This matrix formulation of the index is suitable for the decomposition along different dimensions we use in our analysis. Consider a city with n neighborhoods. Let $\mathbf{p} = (p_1, \ldots, p_n)^T$ be the $n \times 1$ vector of neighborhood poverty headcount ratios sorted in decreasing order and $\mathbf{s} = (s_1, \ldots, s_n)^T$ be the $n \times 1$ vector of the corresponding population shares. $\mathbf{1}_n$ being the $n \times 1$ vector with each element equal to 1, $\mathbf{P}$ is the $n \times n$ skew-symmetric matrix:

$$\mathbf{P} = \frac{1}{\bar{p}} \left(\mathbf{1}_n \mathbf{p}^T - \mathbf{p} \mathbf{1}_n^T \right) = \begin{bmatrix} \frac{p_1 - p_1}{\bar{p}} & \cdots & \frac{p_n - p_1}{\bar{p}} \\ \vdots & \ddots & \vdots \\ \frac{p_1 - p_n}{\bar{p}} & \cdots & \frac{p_n - p_n}{\bar{p}} \end{bmatrix}, \tag{1}$$

where $\bar{p}$ is the poverty headcount ratio for the whole city. Let $\mathbf{S} = diag\{\mathbf{s}\}$ be the $n \times n$ diagonal matrix with diagonal elements equal to the population shares in $\mathbf{s}$, and $\mathbf{G}$ be a $n \times n$ G-matrix (a skew-symmetric matrix whose diagonal elements are equal to 0, with upper diagonal elements equal to -1 and lower diagonal elements equal to 1) [7]. The matrix formulation of the Gini index of urban poverty is

$$G(\mathbf{s},\mathbf{p}) = \frac{1}{2} tr\left(\tilde{\mathbf{G}}\mathbf{P}^T\right), \tag{2}$$

where the matrix $\tilde{\mathbf{G}} = \mathbf{SGS}$ is the weighting G-matrix, a generalization of the G-matrix [1, 5].

2.1 The components of the change in concentrated poverty

Assume that the distributions of poor and non-poor individuals between the neighborhoods in a city are observed at different times, notably t and $t+1$. Let $\mathbf{p}_t$ be the $n \times 1$ vector of the t neighborhood poverty headcount ratios sorted in decreasing order and $\mathbf{s}_t$ be the $n \times 1$ vector of the corresponding neighborhood population shares. Let $\mathbf{p}_{t+1}$ be the $n \times 1$ vector of the $t+1$ neighborhood poverty headcount ratios sorted in decreasing order and $\mathbf{s}_{t+1}$ be the $n \times 1$ vector of the corresponding population shares. The change in the degree of concentrated poverty between t and $t+1$ can be measured by the difference between the Gini index in $t+1$ and the Gini index in t [1]:

$$\Delta G = G(\mathbf{s}_{t+1},\mathbf{p}_{t+1}) - G(\mathbf{s}_t,\mathbf{p}_t) = \frac{1}{2} tr\left(\tilde{\mathbf{G}}_{t+1}\mathbf{P}_{t+1}^T\right) - \frac{1}{2} tr\left(\tilde{\mathbf{G}}_t\mathbf{P}_t^T\right). \tag{3}$$

As shown by Andreoli and Mussini [1], equation 3 can be decomposed to separate the components attributable to changes in neighborhood population shares, ranking of neighborhoods by poverty level and disparities between neighborhood poverty headcount ratios. To decompose ΔG some additional definitions and notation are needed. Let $\mathbf{p}_{t+1|t}$ be the $n \times 1$ vector of $t+1$ neighborhood poverty headcount ratios sorted in decreasing order of the respective t neighborhood poverty headcount ratios, and $\mathbf{B}$ be the $n \times n$ permutation matrix rearranging the elements of $\mathbf{p}_{t+1}$ to obtain $\mathbf{p}_{t+1|t}$. Let $\lambda = \bar{p}_{t+1}/\bar{p}_{t+1|t}$ be the ratio of the actual $t+1$ poverty headcount ratio in the whole city to the fictitious $t+1$ poverty headcount ratio which is the weighted average of $t+1$ neighborhood poverty headcount ratios where the weights are the corresponding population shares in t. The elements of $\mathbf{P}_{t+1|t} = \left(1/\bar{p}_{t+1|t}\right)\left(\mathbf{1}_n\mathbf{p}_{t+1|t}^T - \mathbf{p}_{t+1|t}\mathbf{1}_n^T\right)$ are the relative pairwise differences between the neighborhood poverty headcount ratios in $\mathbf{p}_{t+1|t}$. The decomposition of ΔG is

$$\Delta G = \frac{1}{2} tr\left(\mathbf{W}\mathbf{P}_{t+1}^T\right) + \frac{1}{2} tr\left(\mathbf{R}\lambda\mathbf{P}_{t+1}^T\right) + \frac{1}{2} tr\left(\tilde{\mathbf{G}}_t\mathbf{D}^T\right) = W + R + D, \tag{4}$$

where $\mathbf{W} = \tilde{\mathbf{G}}_{t+1} - \lambda\tilde{\mathbf{G}}_{t|t+1}$, $\mathbf{R} = \tilde{\mathbf{G}}_{t|t+1} - \mathbf{B}^T\tilde{\mathbf{G}}_t\mathbf{B}$ and $\mathbf{D} = \mathbf{P}_{t+1|t} - \mathbf{P}_t$. In equation 4, W is the component measuring the change in concentrated poverty due to changes in the distribution of population between neighborhoods. R is the re-ranking component that is greater than zero when at least two neighborhoods exchanged their ranks in the distribution of neighborhood poverty headcount ratios between t and $t+1$. D is the component measuring the change in relative disparities between neighborhood poverty headcount ratios. D is positive (negative) when relative disparities between neighborhood poverty headcount ratios increased (decreased) over time [1].

2.1.1 The role of the change in poverty incidence

Since component D would not reveal changes in neighborhood poverty headcount ratios if they all changed in the same proportion, this component is split into two further terms: one measuring the change in the poverty headcount ratio in the whole city, the second measuring the changes in relative disparities between neighborhood poverty headcount ratios by assuming that the poverty headcount ratio in the city is unchanged between t and $t+1$. Let c be the change in the poverty headcount ratio in the city by assuming that neighborhood population shares are unchanged over time:

$$c = \frac{\bar{p}_{t+1|t} - \bar{p}_t}{\bar{p}_t}. \tag{5}$$

Let $\mathbf{p}^c_{t+1|t} = \mathbf{p}_t + c\mathbf{p}_t$ be the vector of neighborhood poverty headcount ratios we would observe in $t+1$ if the change in every neighborhood poverty headcount ratio was equal to c in relative terms. Vector $\mathbf{p}_{t+1|t}$ can be expressed as

$$\mathbf{p}_{t+1|t} = \mathbf{p}^c_{t+1|t} + \mathbf{p}^\delta_{t+1|t}, \tag{6}$$

where the elements of vector $\mathbf{p}^\delta_{t+1|t}$ are the element-by-element differences between vectors $\mathbf{p}_{t+1|t}$ and $\mathbf{p}^c_{t+1|t}$. Since $\mathbf{p}^c_{t+1|t} = \mathbf{p}_t + c\mathbf{p}_t$, $\mathbf{p}_{t+1|t}$ can be rewritten as

$$\begin{aligned} \mathbf{p}_{t+1|t} &= \underbrace{\mathbf{p}_t + \mathbf{p}^\delta_{t+1|t}}_{\mathbf{p}^e_{t+1|t}} + c\mathbf{p}_t \\ &= \mathbf{p}^e_{t+1|t} + c\mathbf{p}_t, \end{aligned} \tag{7}$$

where the elements of $\mathbf{p}^e_{t+1|t}$ account for disproportionate changes in neighborhood poverty headcount ratios from t to $t+1$, as $\mathbf{p}^e_{t+1|t}$ would equal $\mathbf{p}_t$ if there were no disproportionate changes in neighborhood poverty headcount ratios. Given equations 5 and 7, matrix $\mathbf{P}_{t+1|t}$ can be written as

$$\begin{aligned} \mathbf{P}_{t+1|t} &= \left(1/\bar{p}_{t+1|t}\right)\left(\mathbf{1}_n \mathbf{p}^T_{t+1|t} - \mathbf{p}_{t+1|t}\mathbf{1}^T_n\right) \\ &= \frac{1}{1+c}\mathbf{P}^e_{t+1|t} + \frac{c}{1+c}\mathbf{P}_t. \end{aligned} \tag{8}$$

Since matrix $\mathbf{D}$ in equation 4 is obtained by subtracting $\mathbf{P}_t$ from $\mathbf{P}_{t+1|t}$, $\mathbf{D}$ can be rewritten as

$$\mathbf{D} = \mathbf{P}_{t+1|t} - \mathbf{P}_t \tag{9}$$

$$
\begin{aligned}
&= \frac{1}{1+c}\mathbf{P}^e_{t+1|t} + \frac{c}{1+c}\mathbf{P}_t - \mathbf{P}_t \\
&= \underbrace{\left(\frac{1}{1+c}\right)}_{C}\underbrace{\left(\mathbf{P}^e_{t+1|t} - \mathbf{P}_t\right)}_{\mathbf{E}} \\
&= C\mathbf{E}.
\end{aligned}
$$

By replacing $\mathbf{D}$ in equation 4 with its expression in equation 9, the decomposition of the change in concentrated poverty becomes

$$
\Delta G = \frac{1}{2}tr\left(\mathbf{W}\mathbf{P}^T_{t+1}\right) + \frac{1}{2}tr\left(\mathbf{R}\lambda\mathbf{P}^T_{t+1}\right) + C\frac{1}{2}tr\left(\tilde{\mathbf{G}}_t\mathbf{E}^T\right) = W + R + CE. \tag{10}
$$

C in equation 10 measures the change in the poverty headcount ratio for the whole city, and E captures the change in relative disparities between neighborhood poverty headcount ratios once the effect of the change in the proportion of poor people in the city has been removed. In other words, E is a "pure" component of disproportionate change between neighborhood poverty headcount ratios.

2.2 Spatial decomposition of the change in concentrated poverty

The components of the change in concentrated poverty described in Sect. 2.1 can be broken down into spatial components by using the Rey and Smith approach to the spatial decomposition of the Gini index [6]. Building on Andreoli and Mussini [1], the spatial components of ΔG, W, R and E are obtained. Let $\mathbf{N}_t$ be the $n \times n$ spatial weights matrix having its (i, j)-th entry equal to 1 if and only if the (i, j)-th element of $\mathbf{P}_t$ is the relative difference between the poverty headcount ratios of two neighboring neighborhoods, otherwise the (i, j)-th element of $\mathbf{N}_t$ is 0. Using the Hadamard product,[1] the relative pairwise differences between the poverty headcount ratios of neighboring neighborhoods can be selected from $\mathbf{P}_t$:

$$
\mathbf{P}_{N,t} = \mathbf{N}_t \odot \mathbf{P}_t. \tag{11}
$$

Since $\mathbf{P}^e_{t+1|t}$ and $\mathbf{P}_t$ are defined by the ordering of neighborhoods in t, $\mathbf{N}_t$ also selects the relative pairwise differences between neighboring neighborhoods from $\mathbf{P}^e_{t+1|t}$:

$$
\mathbf{P}^e_{N,t+1|t} = \mathbf{N}_t \odot \mathbf{P}^e_{t+1|t}. \tag{12}
$$

Given that $\mathbf{E} = \mathbf{P}^e_{t+1|t} - \mathbf{P}_t$, the Hadamard product between $\mathbf{N}_t$ and $\mathbf{E}$ is a matrix with nonzero elements equal to the elements of $\mathbf{E}$ pertaining to neighboring neighborhoods:

[1] Let $\mathbf{X}$ and $\mathbf{Y}$ be $k \times k$ matrices. The Hadamard product $\mathbf{X} \odot \mathbf{Y}$ is defined as the $k \times k$ matrix with the (i, j)-th element equal to $x_{ij}y_{ij}$.

$$\mathbf{E}_N = \mathbf{P}^e_{N,t+1|t} - \mathbf{P}_{N,t} = \mathbf{N}_t \odot \left(\mathbf{P}^e_{t+1|t} - \mathbf{P}_t\right) = \mathbf{N}_t \odot \mathbf{E}. \tag{13}$$

Let $\mathbf{P}_{N,t+1}$ be the matrix whose nonzero elements are the relative pairwise differences between the poverty headcount ratios of neighboring neighborhoods in $t+1$:

$$\mathbf{P}_{N,t+1} = \mathbf{N}_{t+1} \odot \mathbf{P}_{t+1}. \tag{14}$$

The decomposition of the change in the neighbor component of concentrated poverty is obtained by replacing $\mathbf{P}_{t+1}$ and $\mathbf{E}$ in equation 10 with $\mathbf{P}_{N,t+1}$ and $\mathbf{E}_N$, respectively:

$$\begin{aligned} \Delta G_N &= \frac{1}{2} tr\left(\mathbf{W}\mathbf{P}^T_{N,t+1}\right) + \frac{1}{2} tr\left(\mathbf{R}\lambda\mathbf{P}^T_{N,t+1}\right) + C\frac{1}{2} tr\left(\tilde{\mathbf{G}}_t\mathbf{E}^T_N\right) \\ &= W_N + R_N + CE_N. \end{aligned} \tag{15}$$

$\mathbf{J}_n$ being the matrix with diagonal elements equal to 0 and extra-diagonal elements equal to 1, the matrix with nonzero elements equal to the relative pairwise differences between the $t+1$ poverty headcount ratios of non-neighboring neighborhoods is

$$\mathbf{P}_{nN,t+1} = (\mathbf{J}_n - \mathbf{N}_{t+1}) \odot \mathbf{P}_{t+1}. \tag{16}$$

The matrix selecting the elements of $\mathbf{E}$ related to the pairs of non-neighboring neighborhoods is

$$\mathbf{E}_{nN} = (\mathbf{J}_n - \mathbf{N}_t) \odot \mathbf{E}. \tag{17}$$

The decomposition of the change in the non-neighbor component of concentrated poverty is obtained by replacing $\mathbf{P}_{t+1}$ and $\mathbf{E}$ in equation 10 with $\mathbf{P}_{nN,t+1}$ and $\mathbf{E}_{nN}$, respectively:

$$\begin{aligned} \Delta G_{nN} &= \frac{1}{2} tr\left(\mathbf{W}\mathbf{P}^T_{nN,t+1}\right) + \frac{1}{2} tr\left(\mathbf{R}\lambda\mathbf{P}^T_{nN,t+1}\right) + C\frac{1}{2} tr\left(\tilde{\mathbf{G}}_t\mathbf{E}^T_{nN}\right) \\ &= W_{nN} + R_{nN} + CE_{nN}. \end{aligned} \tag{18}$$

Given equations 15 and 18, the spatial decomposition of the change in concentrated poverty is

$$\Delta G = W_N + W_{nN} + R_N + R_{nN} + C\left(E_N + E_{nN}\right). \tag{19}$$

3 The dynamics of concentrated poverty in American cities

We use information on income and population distributions within U.S. metro areas over the 1980-2016 period from the U.S. Census Bureau database. Information about population counts, income levels and family composition at a very fine spatial grid is taken from the decennial census Summary Tape File 3A. Census tracts are the spatial units of observation, and poverty headcount ratios at the federal poverty line provided by the U.S. Census Bureau are calculated. The 1980-2016 period is divided into five sub-periods to observe the dynamics of each component of the change in concentrated poverty. Since the changes in the population distribution within cities played a minor role in the change in concentrated poverty, we focus our attention on the components measuring the re-ranking effect (R) and the effect of the disproportionate change between census tract poverty headcount ratios (E) for the largest three American metro areas: New York, Los Angeles and Chicago. Concentrated poverty decreased in each of the three metro areas during the period considered, with the largest reduction in Chicago ($\Delta G = -0.11088$) where concentrated poverty ($G = 0.54921$) was greater than in the other two cities in 1980. The degree of concentrated poverty was 0.49669 in New York and 0.41069 in Los Angeles in 1980. Figure 1 shows the spatial decomposition of E. The poverty headcount ratios of non-neighboring census tracts in Chicago have become less unequal, especially during the decade from 2000 to 2010. The decrease in disparities between the poverty headcount ratios of non-neighboring census tracts has been less pronounced in New York and Los Angeles. The decrease in disparities between the poverty headcount ratios of neighboring census tracts in New York has been greater than in the other two cities.

Figure 2 shows the spatial components of the re-ranking effect. The largest re-ranking effect occurred between non-neighboring census tracts in Chicago during the 2000-2010 sub-period. This re-ranking effect partly offset the effect of the reduction in inequality between the poverty headcount ratios of non-neighboring cen-

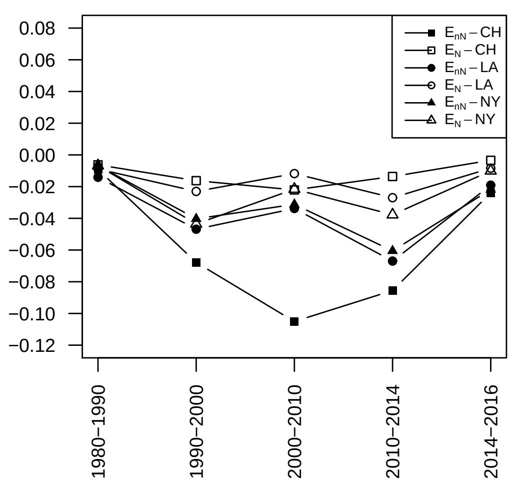

Fig. 1 Spatial components of E in Chicago (CH), Los Angeles (LA) and New York (NY) in the 1980-2016 period.

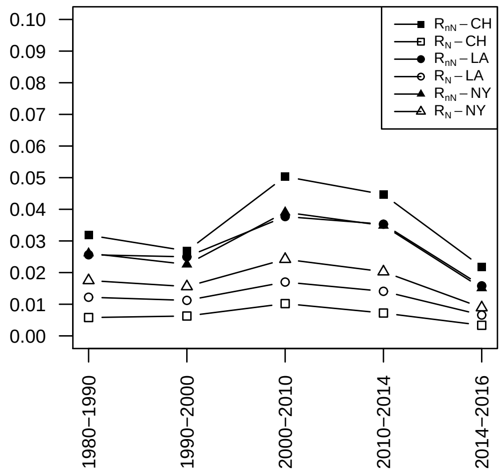

Fig. 2 Spatial components of R in Chicago (CH), Los Angeles (LA) and New York (NY) in the 1980-2016 period.

sus tracts in the city in that decade, especially in view of the increase in poverty incidence in the city ($C = 0.79812$) that weakened the effect of the reduction in inequality between census tract poverty headcount ratios.

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