Sovereign co-risk measures in the Euro Area
Dipendenza del rischio sovrano tra paesi dell'area Euro

Giuseppe Arbia, Riccardo Bramante, Silvia Facchinetti, Diego Zappa

Abstract We propose a method to extract significant risk interactions between Countries adopting the Graphical Lasso algorithm, used in graph theory to sort out the spurious effect of common components. In this context, the major issue is the definition of the penalization parameter. We propose a search algorithm aimed at the best separation of the variables (expressed in terms of conditional dependence) given an a priori desired partition. The case study focuses on Sovereign Bond Yields over the period 2009–2017. The proposed algorithm is used in systemic risk estimation of the Euro area sovereigns.

Abstract *L'algoritmo Glasso è noto nella letteratura associata alla teoria dei grafi per filtrare correlazioni spurie, se presenti. Il principale problema di questo metodo è la calibrazione del parametro di penalizzazione. In questo lavoro viene proposto un algoritmo volto a trovare la miglior separazione delle variabili in esame, in termini di indipendenza condizionale, assegnata una partizione ottimale definita a priori. L'applicazione esamina la relazione tra rendimenti dei bond sovrani nell'area Euro nel periodo 2009–2017 al fine di verificare l'assunto della separazione o della interdipendenza tra paesi Core e Periferici.*

Key words: Graphical Lasso algorithm, systemic risk, network dependence

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1 Introduction

We propose an original approach based on Graphical Lasso (GLasso; see Friedman et al. 2008) to investigate government bond yield data interactions from a systemic risk perspective.

The recent debt crisis in the Euro Area has turned researchers' attention to sovereign default risk measurement of. Specifically, the degree of co–movements of Sovereign bond spreads among countries can help us to understand how correlations of default probabilities – as measures of perceived country risk – evolve over time and are diffused in space. For both descriptive purposes and quality picture representation, Figure 1 depicts the corresponding bond trajectories from January 2009 to October 2017, focusing only on the 4 major developed countries in the EU (Italy, Spain, Germany and France).
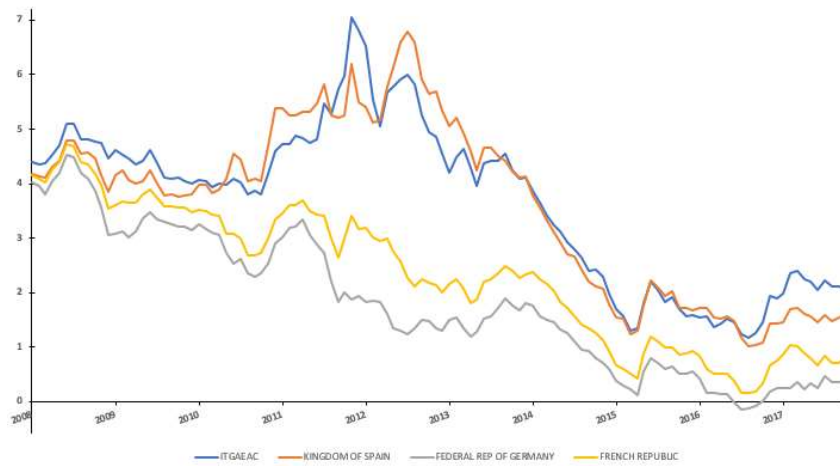

Fig. 1. Ten–year Sovereign Bond Yields

The pattern of the series in the graph unambiguously shows the effect of the crisis on Italy and Spain. Bond yields have dramatically risen from 2011 to 2013, a period affected by wide changes in global risk aversion. In particular, from 2010 yields began moving upwards, continuing to widen sharply in 2011. The sharp decline started in the second half of 2012 due to the European Central Bank policies, with a subsequent stabilization around a roughly flat trend.

In this paper, we propose the use of GLasso to focus the study only on relevant sovereign risk co–movements within the Euro Area assuming the conditional independence between Core and Peripheral Countries (ECB, 2016)[1], following the approach described in Arbia et al. (2018).

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1   For an application of GLasso in the financial field see, for example, Goto and Xu, 2015.

More specifically, the aim of this paper is twofold. First of all, our contribution focuses on the choice of the penalty parameter within the GLasso framework. As this choice is in most cases subjective and expertise-driven, we propose to modify the calibration search algorithm in Friedman et. al (2008) by searching for the minimum of the absolute difference between the Government Bond yield returns expected precision matrix and its estimate after penalization. Secondly, we analyse cross-country contagion effects by investigating – in a rolling framework – the characteristics of the degree of connectivity of the examined countries, in the graph obtained after penalization.

The rest of the paper is organized as follows. Section 2 reports some details about the GLasso algorithm and the procedure developed to calibrate the penalization parameter. In light of the Sovereign debt crisis, the application proposed in Section 3 refers to Euro Zone systemic risk analysis. Conclusions follow.

2 Methodology

Let $X=\{X_1, X_2,..., X_p\}$ be a p–multivariate random variable. Let Σ and $\Theta=\Sigma^{-1}$ be its covariance and precision matrix, respectively. It may be shown that Θ is proportional to the partial correlation matrix and so Θ can be effectively used to characterize the interrelationship of the variable of interest through the associated graph (Edwards, 2000).

The key role played by Θ justifies the many approaches proposed to estimate it efficiently and robustly with respect to abnormal deviations (see e.g. Ledoit & Wolf, 2012). A recent approach is represented by the GLasso algorithm which uses a regularization framework to estimate the covariance matrix under the assumption that its inverse is sparse. The aim of GLasso is to estimate Σ or Θ by removing the elements that likely denote spurious correlations. The way to achieve this is by introducing a penalization into the maximum likelihood estimation of the precision matrix using an L_1 penalty function over nonnegative definite matrices Θ:

$$\arg\max_{\Theta>0}\{\log\det\Theta - \mathrm{tr}(S\Theta) - \lambda\|\Theta\|_1\} \qquad (1)$$

where $\|\Theta\|_1$ is the L_1 norm of Θ, S is the empirical covariance matrix and λ a scalar parameter that controls the size of the penalty. The smaller the value of λ is, the higher will be the degree of dependence between the variables and the density of the graph.

According to the work of Banerjee et al. (2008), it is possible to define the dual of sparse maximum likelihood problem in (1) as

$$\arg\max_{W}\{\log\det W : \|W - S\|_\infty \leq \lambda\} \qquad (2)$$

where $W=S+U$ and U is a symmetric matrix that allow to represent $\|\Theta\|_1$ as

$$\|\Theta\|_1 = \max_{\|U\|_\infty \leq 1} \mathrm{tr}[\Theta U]$$

The dual problem (2) estimates the covariance matrix while the primal problem (1) its inverse. Moreover, log function is a monotone increasing function, thus we can also use the equivalent problem removing the log.

For further details on GLasso algorithm see Friedman et al. (2008), Banerjee et al. (2008) and Witten et al. (2011), among other.

The penalized maximum likelihood estimation for Σ can be computed $\forall \lambda \geq 0$.

Let Θ_H and $\widehat{\Theta}_\lambda$ represent the expected/desired precision matrix and the estimate of Θ after penalization, respectively. We define a flexible search algorithm to solve:

$$\min_{\lambda} \left\| \Theta_H - \widehat{\Theta}_\lambda \right\|_1 \tag{3}$$

It allows us to define a conditional dependence structure without the constraint to get a solution that exactly matches that structure. Differently from standard GLasso constraints procedures, the solution to Equation (3) allows some elements of Θ to be different from zero.

3   Empirical evidence from Euro Area Sovereign bond markets

Graphical Lasso algorithm has been applied to 10-year Government bond yields monthly data covering 17 countries in the Eurozone. This includes five Peripheral Countries, five Core Countries and seven "Other" remaining countries, which subsequently (except for Finland) adopted the Euro[1]. Data spans the period from January 2009 to October 2017; covariance and precision matrices, along with the corresponding λ parameters, are estimated over three rolling windows of size 150, 200 and 300 days.

To reflect the evolution of systemic risk and assess the degree of connection among the Core, Peripheral and "Other" countries we dynamically apply the GLasso algorithm to estimate at time t the precision block matrix Θ_t as follows:

$$\Theta_t = \begin{bmatrix} {}_{PP}\Theta_t & {}_{PC}\Theta_t & {}_{PO}\Theta_t \\ {}_{CP}\Theta_t & {}_{CC}\Theta_t & {}_{CO}\Theta_t \\ {}_{OP}\Theta_t & {}_{OC}\Theta_t & {}_{OO}\Theta_t \end{bmatrix}$$

where t is the time index and subscripts P, C and O indicate Peripheral, Core and remaining (Other) countries, respectively.

Our aim is to apply Equation (3) to extract graphs with a good balance between sparsity and density, avoiding the selection of large λ, which may artificially inflate sparsity in the precision matrix. Specifically, the goal is a zero PC block in Θ_t e.g., the Peripheral and Core Countries are conditionally independent and no co–risk between the two groups is considered. The dynamics of λ_t according to the results

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1   Luxembourg is excluded from the analysis since no data are available.

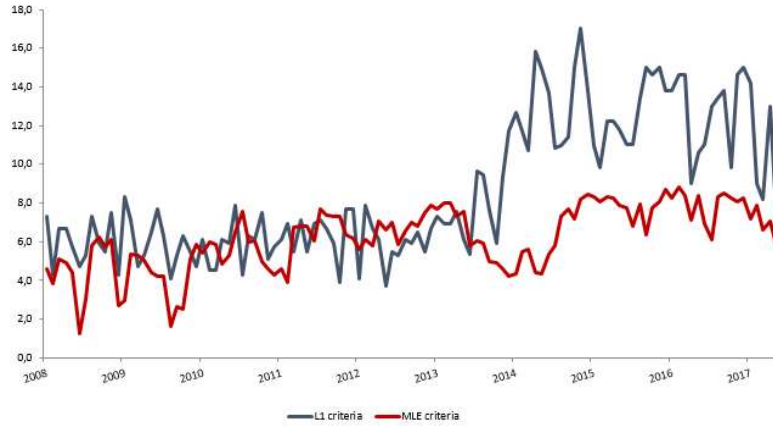obtained by the application of Equation (3) is reported in Figure 2, by comparing the MLE and L_1 estimates.

Fig. 2. λ_t comparison: *MLE* versus L_1 criterion

This series give evidence of a sharp increase in λ_t values – when the L_1 criterion is considered – during the mounting sovereign debt crisis. Moreover, while 2014 seems to represent a lower boundary of the crisis, the aftermath of a continued erratic pattern indicates that the sovereign debt crisis seems far from a final resolution.

As an example, Figure 3 shows the graphs obtained by using the best solution to Equation (3): that is, the application of MLE (Figure 3B) and L_1 criterion (Figure 3C), referring to the last rolling time window (October 2017).
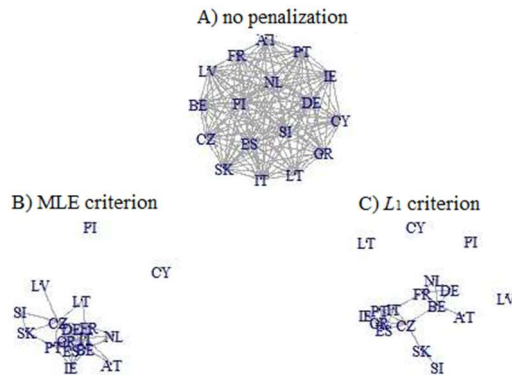

Fig. 3. Network with A) no penalization, optimal B) MLE and C) L1 values

The graphs map out the active connections obtained in the two frameworks, giving evidence of the capabilities of the L_1 criterion to filter out the spurious effects of common components. In particular, the reinforced interconnectedness within the *PP*

countries after the crisis are shown and the evidence of only an active contagion transmission channel between *PP* and *CC* groups referred to the country pair Italy-France is given. MLE solution, on the other hand, lacks of a clear pattern.

4 Conclusions

This paper exploited the Graphical Lasso algorithm to extract significant correlations in Government bond yields returns series. A calibration criterion to identify the best regularization parameter along time and cross–sectionally has been proposed.

Empirical evidences from the Euro area show that the proposed method allows to extract the relevant systemic risk contributions and identify the most interconnected nodes (countries), thus lowering the network dimension and isolating spurious relationships. We also show that the penalization parameter can be used as an indicator of the intensity of the crisis and that a better description of the relationships between Peripheral and Core Countries is obtained using the L_1 criterion instead of the *MLE* one.

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