How can we compare rankings that are expected to be similar? An example based on composite well being indicators.

Confronti tra graduatorie simili. Un esempio basato su indicatori compositi di benessere.

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Abstract We compare 5 different well-being rankings derived from the following composite indicators: Human Development Index, Inequality Adjusted Human Development Index, Legatum Prosperity Index, Good Nation Index, Development Sustainable Goals Index.
Abstract *Confrontiamo tra loro 5 diverse graduatorie di benessere basate sui seguenti indicatori composti: Human Development Index, Inequality Adjusted Human Development Index, Legatum Prosperity Index, Good Nation Index, Development Sustainable Goals Index.*
Key words: well being, concordance, local concordance

1 Well being indicators and derived rankings

The most widely-used and well-known composite indicator of well being is the Human Development Index (HDI). It is a summary measure of average achievement in key dimensions of human development: *a long and healthy life,*

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being knowledgeable and have a decent standard of living [3]. The HDI is the geometric mean of normalized indices (life expectancy, education, Gross National Income) for each of the three dimensions. However this index does not account for disparities (inequalities) within each dimension across the population; thus United Nations Development Programme also computes an Inequality adjusted HDI (IHDI). The IHDI combines a country's average achievements in health, education and income with how those achievements are distributed among country's population by "discounting" each dimension's average value according to its level of inequality.[2]

Other well being indicators available for a large number of countries worldwide are: the Legatum Prosperity Index, the Good Country Index, the Sustainable Development Goals Indicator. The Legatum Prosperity Index (LPI) [8], based on 104 indicators , is an aggregation of nine sub-indexes: (1) economic quality, (2) business environment, (3) governance, (4) education, (5) health, (6) safety & security, (7) personal freedom, (8) social capital, (9) natural environment.

The Good Country Index (GCI) [1] is based on 35 indicators related to the following 7 dimensions: (i) Science, Technology & Knowledge; (ii) Culture; (iii) International Peace and Security; (iv) World Order; (v) Planet and Climate; (vi) Prosperity and Equality; and (vii) Health and Wellbeing.

The Sustainable Development Index (SDGI) is based on several normalized indicators for each of the 17 Sustainable Development Goals (see [6] for the list of indicators and goals).

A careful insight into these different indicators goes beyond the standards allowed for our communication. Otoiou et al [5] explore whether the variables used in computing three of the most widely known indicators of well-being and social progress, the HDI, LPI, and Happy Planet Index, can be used to develop a relevant cluster structure, which can then be used to assess the validity and reliability of the country rankings obtained by these indicators. Among other comments and conclusions, they argue that the optimal cluster structure is very close to HDI country rankings. Making moves from the assumption that all the quoted indicators are well being indicators, and that what receives attention from media and policy makers are the country rankings of these different composite indicators, our interest focuses on the ranking comparison. How can we measure the similarities or the distances or the agreement between the different rankings? Is one of these rankings to be preferred with respect to the others? Are our comparisons in line with Otoiou et al.?

[2] A different kind of adjustment has been suggested by Terzi [7]: to correct HDI by means of a Multidimensional Poverty Indicator. This gives rise to the *corrected HDI*. However, MPI is available only for a subset of the countries for which HDI is computed so we decided not to consider MPI; not as a well-being indicator nor as a correction factor.

2 Ranking comparisons

First of all let us examine the correlations among the rankings of the 5 chosen indexes (Good Country [1], Human Development [3], Inequality Adjusted HDI [3], Legatum Prosperity [8], Sustainable Development Goals [6]) for n = 120 countries, for the most recent available years (2017 for GCI and SDGI; 2016 for the others).

	GCI	HDI	IHDI	LPI	SDGI
GCI	1.000	0.843	0.852	0.851	0.836
HDI	0.843	1.000	0.983	0.896	0.939
IHDI	0.852	0.983	1.000	0.888	0.958
LPI	0.851	0.896	0.888	1.000	0.872
SDGI	0.836	0.939	0.958	0.872	1.000

Table 1: correlations among rankings

Also, we have the concordance:

Kendall's $W = 0.913$.

From the correlations and from Kendall's coefficient, the 5 rankings appear to be very close (in particular HDI, IHDI, SDGI).

However what could interest mostly the policy makers are the differences in rankings of the best and of the worst performing countries. For this reason we resort to Top-Down concordance coefficient [4].[3] The Top-Down concordance coefficient is derived by computing Kendall's W not on ranks R_i but instead on Savage scores

$$S_{R_i} = \sum_{j=R_i}^{n} \frac{1}{j}$$

In particular we have a top-down *low* concordance coefficient based on W (W_{TDL}) by substituting each rank R_i with the respective Savage score S_{R_i} when our interest is for the concordance among the lowest ranks, i.e. the top of the distribution; and a top-down *high* concordance coefficient when our interest is for the highest ranks, i.e. to bottom of the distribution. In this case we substitute each rank R_i with the Savage score:

$$S_{n-R_i+1} = \sum_{j=n-R_i+1}^{n} \frac{1}{j}$$

thus obtaining the Top-Down (high) concordance coefficient W_{TDH}. By the way, this is the same as calculating W_{TDL} on the descendent rankings of the objects.

For our rankings we have:

[3] If interested in the agreement among two rankings this could be achieved by means of weighted rank correlation, derived from Spearman's ρ (see for example Dancelli et al [2]).

Top-Down Kendall high $W_{TDH} = 0.892$,
Top-Down Kendall low $W_{TDL} = 0.839$.

Thus the 5 rankings are closer in the tail of the distributions than for the best performing countries.

To have an even closer insight of the concordance in the top or bottom or in the central rankings, we consider a local headcount function. We partition the set of the first $n = 120$ natural numbers (the ranks of the 120 observed countries) in contiguous subsets of fixed size s (for example 10% of the observations; 12 in our case) of consecutive naturals, and count for each subset how many units (countries), for at least one of the $d = 5$ rankings, rank in that subset. A local headcount of $s = 12$, the minimum of its range, means that in the interval spanned by the subset there are exactly s units, whose ranks are thus very close on all indicators. Conversely, a local headcount of $min(n, s.d) = 60$, the maximum of its range, means that in the span of the subset no unit ranks more than once.

The interpretation we suggest for the local headcounts is that they are inverse indicators of local concordance, because the headcounts are smaller when the ranks are closer and bigger when the ranks are spaced; so we derive a local concordance function as the maximum, $min(n, s.d) = 60$, minus the value of the local headcount. Moreover, we calculate the local headcount and the local concordance for each subset of s consecutive naturals in $(1, \ldots, n)$, and associate the value to the central rank of the subset, so that we can plot a smooth curve (Figure 1); the local concordance of the partition subsets are enhanced in red.

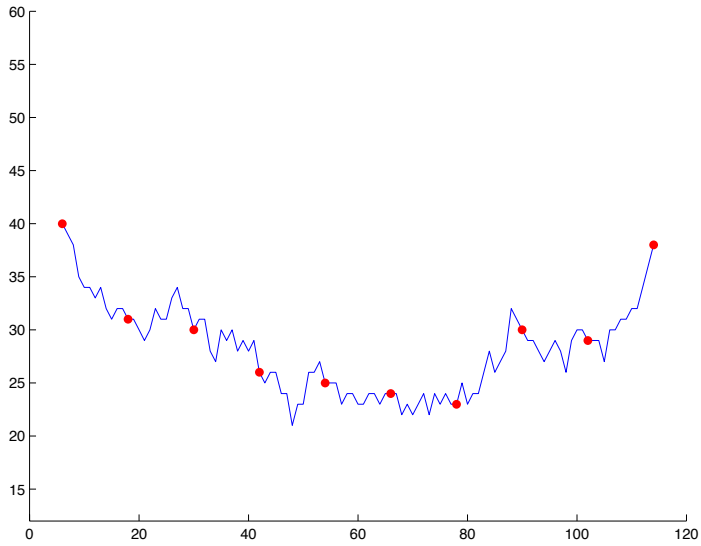

Figure 1: local concordance function

Alternatively we could be interested in finding out which well being indicator best represents all the others.

Let us define a centrality measure $C_{1(J)}$ for each ranking J that counts to how many units it assigns the median rank among the 5. Let us also define a de-centrality measure $C_{2(J)}$ as the sum of the differences between the unit's rank (in the J-th ranking) and the median rank. These values are summarized in Table 2.

Rankings	C_1	C_2
GCI	13	1365
HDI	36	449
IHDI	40	411
LPI	24	1079
SDGI	29	721

Table 2: centrality and de-centrality

Thus the ranking that best represents all the five well-being rankings is IHDI.

The same conclusion appears from Figure 2, in which on the *x*-axis we represent the countries ordered by means of their median rank and on the *y*-axis the ranks and their median.

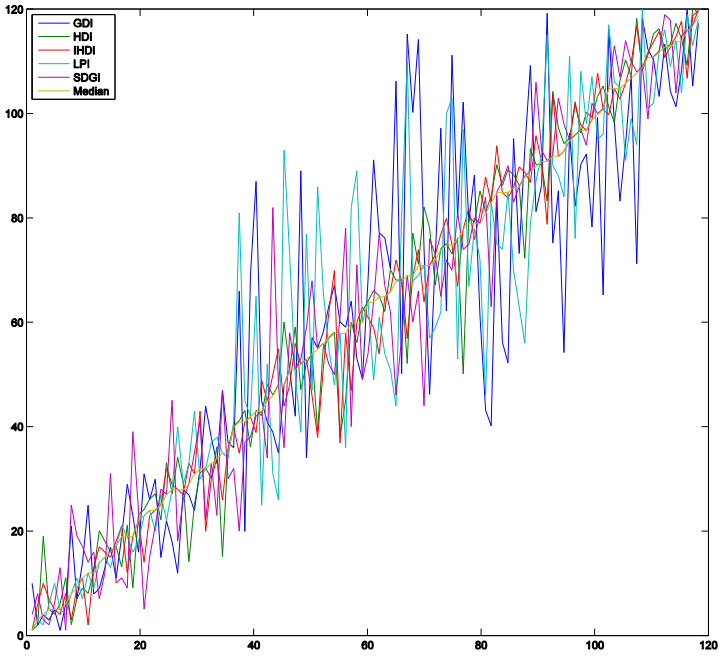

Figure 2: rankings and median by country vs ranking of medians

3 Concluding remarks

Although for the chosen well-being indicators Kendall's coefficient is high (W = 0,913), there are differences among the five rankings. In particular, there are differences among the units ranked in the center of their ranking (as can be seen by means of our local concordance function) and asymmetries in concordance among highest and lowest ranks (as can be seen by computing top-down concordance coefficients). From Figure 2 we can draw similar considerations: the greatest variability among ranks is in the centre of the plot; moreover, it can be seen that the farthest-away-from-the-others ranking is the GCI based ranking, whereas the most intermediate is based on IHDI, and this conclusion is reinforced by our centrality and de-centrality measures in Table 2. Overall, our findings seem consistent with Otoiu's findings [5].

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