Systemic events and diffusion of jumps

Eventi sistemici e diffusione dei jumps

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Abstract We propose two indexes informative of the cross-sectional diffusion of jumps from the analysis of a very large dataset of high-frequency returns that is not common in the literature. The two indexes have important implications in terms of asset pricing, as they capture part of the variability in stock returns that is not explained by the factors of the standard capital asset pricing model.

Abstract *Attraverso l'analisi di un ampio dataset di rendimenti ad alta frequenza, non comune nella letteratura, proponiamo due indici che forniscono informazioni sulla diffusione dei jumps tra le varie societá analizzate. Tali indici sono particolarmente informativi in termini di pricing, dato che incorporano una parte della variabilitá dei rendimenti che non é spiegata dai fattori che caratterizzano il tradizionale capital asset pricing model.*

1 Data and jump detection

We analyse co-jumps involving a relatively large number of stocks using a huge dataset of high-frequency returns, which is not common in the literature. The

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database includes the $N = 3,509$ assets belonging to the basket of the Russell 3000 index for the period January 2, 1998—June 5, 2015.[1] The stock prices are sampled at a frequency of 1 minute from 09:30 a.m. to 04:00 p.m. for each of the 4,344 business days.[2] As a result, we record at the t-th day $M = 390$ 1-minute closing prices for each stock, denoted as $p_{t,i}$, for $t = 1, ..., T$ and $i = 1, ..., M$. Following a common practice—see [3] and [6], among others—we discard the first 5 minutes of each day to avoid potentially erratic price behaviour resulting from market opening.

[4] note that very high-frequency data are mostly composed of market microstructure noise and suggest to use a 5-minute frequency to mitigate microstructure effects. As a result, our empirical analysis builds on 5-minute returns, that we obtain by aggregating the original 1-minute returns.[3] To cope with market liquidity conditions, we restrict the attention on stocks with a sufficient number of non-null intraday returns to obtain accurate estimates of integrated volatility and jumps. In particular, we implement testing methods, described below, under the condition that, on a given trading day, the percentage of non-zero intraday returns is greater than or equal to 75%. In contrast, we treat the days on which the percentage of non-null returns is lower than 75% as days where no jumps occur. With some abuse of wording, we define assets with more than 25% of intradaily returns equal to zero as illiquid.

We use the C-Tz test proposed by [5] to identify the presence of jumps within each trading day in a cross-section of Russell 3000 constituents. Notably, the C-Tz test provides greater power than other tests based on multipower variation (see [5]). Following [2], we implement the test after standardising the returns to correct for volatility periodicity. Therefore, we improve the detection of small jumps during low volatility periods and reduce spurious detections of jumps at high volatility times. We detect the presence of jumps at both daily and intradaily levels. Then, we also gain knowledge about the location of jumps during the day. We highlight that only a few works use non-parametric tests to explicitly detect intraday jumps.

2 Common jumps

We analyse the cross-sectional diffusion of jumps by using intradaily returns with a 5-minute frequency. By using the co-exceedance rule of [6], we first implement the C-Tz test at the significance level $\alpha = 0.01\%$ to detect intraday jumps. Then, we compute the following variable:

$$C_{t,i} = \sum_{j=1}^{N} \mathbb{I}\{\text{Jump}_{t,i,j} > 0\} \begin{cases} \geq 2 & \text{Co-jump} \\ \leq 1 & \text{Single jump} \end{cases} \tag{1}$$

[1] The dataset is provided by Kibot, and the details are available at http://www.kibot.com. Our dataset includes also dead stocks or stocks that are no longer included in the Russell 3000 index.

[2] The original number of business days in our sample is equal to 4,384. We discard 40 days for which we observe particular tight liquidity conditions.

[3] The results obtained with other frequencies are available upon request.

to verify whether two or more assets record a jump in the same interval, where $\mathbb{I}$ is an indicator function taking the value of 1 when a jump is detected for asset j ($j = 1, \ldots, N$, $N = 3509$) at the intraday interval i ($i = 1, \ldots, 77$ using 5 minutes intervals) on day t ($t = 1, \ldots, 4344$), and the value of 0 otherwise.

Notably, we checked that the C-Tz test is better able to detect common jumps than other common tests in the literature, such as the s-BNS test of [1]. It also highlights structural changes from 2001. Interestingly, we observed that around lunchtime co-jumps tend to increase, whereas individual jumps decrease. Several studies report the existence of a U-shaped pattern for the average volume of traded stocks and, in particular, relatively light trading in the middle of the day. The co-jump intraday pattern, with larger detection in the middle of the day, supports such evidence.[4]

Table 1 Asset jumps and market jumps. The table reports the number of days in which we observe at least one RUA jump (N_{RUA}), the amount of days in which we observe at least one intraday jump (N_j) or one co-jump (N_{cj}) in the constituents of the Russell 3000 and the days with both a jump in the index and a jump ($N_{RUA} \cap N_j$) or a co-jump ($N_{RUA} \cap N_{cj}$) in the underlying assets. These statistics are computed for the period January 1998—June 2015. We consider three observation intervals—1, 5 and 11 minutes. The results are obtained by implementing the C-Tz test.

Frequency	N_{RUA}	N_j	$N_{RUA} \cap N_j$	N_{cj}	$N_{RUA} \cap N_{cj}$
1 min	1,512.00	4,119.00	1,418.00	3,858.00	1,345.00
5 min	176.00	4,333.00	175.00	4,109.00	172.00
11 min	57.00	4,344.00	57.00	4,269.00	57.00

Jumps of individual stocks may affect the entire market. For instance, market-level news causing co-jumps of individual stocks might also be reflected in jumps of market portfolios. Further, co-jumps of stocks involving the market index can be seen as non-diversifiable events, with important implications for portfolio selection and hedging. Here, we define as systematic the co-jumps that occur simultaneously with a jump in the market index. Likewise, we define as non systematic the co-jump events detected from single-asset co-jumps but not reflected in a jump at the market index level. In short, we label as RUA jumps those that occur on the Russell 3000 index (RUA), our proxy for the market index. Table 1 shows jump days for the market index, jump and co-jump days in the underlying assets and the number of jump and co-jump days that are also RUA jump days. As a robustness check, Table 1 reports the results for three observation intervals, that is, 1, 5 and 11 minutes. Interesting findings emerge from Table 1. First, jump (N_j) and co-jump (N_{cj}) days are positively related to the interval length, whereas the opposite holds for the RUA index (N_{RUA}). Second, the number of days with at least one intraday co-jump is always lower than the corresponding number of days with at least one intraday jump.

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[4] Tables and figures displaying such evidences, that we omit here for the sake of space, are available upor request.

Nevertheless, it still takes relevant values: from a minimum of 3,858 days (89% of the sample days) to a maximum of 4,269 days (98% of the sample days). Thus, we observe a co-jump for nearly each day of the sample. We stress that the definition of co-jumps is not particularly restrictive and, thus, we might have co-jumps involving only a small number of assets. Third, columns 4 and 6 of Table 1 present the intersections between jump days in the market index and jump or co-jumps days detected in the underlying assets. $N_{RUA} \cap N_j$ and $N_{RUA} \cap N_{cj}$ are useful to evaluate the capability of RUA to reflect cross-sectional jump events. Starting from the evidence that the majority of jumps and co-jumps do not occur simultaneously with jumps in the index, it is possible to deduce that the jumps in the index are not really informative of the presence of jumps and co-jumps in the cross-section. Since we are aggregating results for daily frequency, outcomes derived from intraday intervals would show even fewer intersections.

3 Multiple co-jumps, diffusion indexes and pricing

We now define as multivariate jump (or MJ) the subset of co-jumps such that:

$$MJ_{t,i} = \begin{cases} \sum_{j=1}^{N} \mathbb{I}\{\text{Jump}_{t,i,j} > 0\} & \text{if } \sum_{j=1}^{N} \mathbb{I}\{\text{Jump}_{t,i,j} > 0\} \geq K \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$

where $K > 2$.

On the basis of (2), we build two indexes: a daily diffusion index (or DID) and an intraday diffusion index (or DII). The DID, for each day from January 2, 1998 to June 5, 2015, equals the largest number of stocks simultaneously jumping within the day. Note that the index might also take a zero value when no MJ occurs in a given day. The DII, in contrast, has an intradaily frequency of 77 observations per day. Each observation points out the number of stocks involved in a multivariate jump, if present, and 0 otherwise. The aim is to analyse the pricing implications of multivariate jumps by extending the standard capital asset pricing model (CAPM)—see [9], [7] and [8]. For the DID (a daily index), we estimate the CAPM and our two-factor model, respectively defined as follows:

$$R_{t,j} - R_{t,F} = \alpha_j + \beta_j MKT_t + e_{t,j}, \tag{3}$$

$$R_{t,j} - R_{t,F} = \alpha_j + \beta_j MKT_t + \beta_{DID,j} DID_t + e_{t,j}, \tag{4}$$

where $R_{t,j}$ is the daily return of the j-th asset, $R_{t,F}$ is the risk-free return that we record from the Kenneth R. French data library, DID_t is the daily diffusion index computed using the C-Tz test and $e_{t,j}$ is a zero-mean residual; $MKT_t = (R_{t,M} - R_{t,F})$ is the excess return on a capitalisation-weighted stock market portfolio, where $R_{t,M}$ is the daily RUA Index return.

Table 2 shows the statistical significance of estimated β_{DID} (denoted as $\hat{\beta}_{DID}$) along with the variations in the R_{adj}^2 values we obtain, including the diffusion in-

dex in the CAPM model. The second column of Table 2 reports the percentage of $\hat{\beta}_{DID}$ with absolute t-statistic greater than 1.645 (10% significance level), using different time windows. Considering the full sample, January 2, 1998—June 5, 2015, we observe that 10% of $\hat{\beta}_{DID}$s are statistically significant. This suggests that the diffusion index could be an important risk factor in asset pricing. Table 2 also reports results for different sub-periods, highlighting how the relevance of DID changes over time and, in particular, focusing on economic crises. DID slopes are significantly different from zero in a relevant number of cases for all sub-periods, with values larger than the full-sample regression. Moreover, it appears that DID slopes are more frequently significant during the pre-2008 economic crisis, namely from 2002 until 2006. The analysis of R-squared highlights the ability of DID to capture part of the variation in stock returns not explained by the traditional market factor. Table 2 reports information on the variations in the R^2_{adj} values we obtain, including the diffusion index in the CAPM model. Min and Max correspond, respectively, to the minimum and maximum difference values, while $Q(0.25)$, Median and $Q(0.75)$ show the values for the first, second and third quartiles of the R^2_{adj} variation. Even if the majority of the variations are negative, the third and fourth quartiles suggest that many variations are positive and larger in absolute value with respect to negative variations. Increases are particularly pronounced during the years 2002—2006 and 2012—2015.

We now move our focus to intraday data. Similar to the daily case, we run monthly regressions using 5-minute data to study how DII helps in explaining stock returns. Our two-factor model for intraday data is:

$$R_{t,i,j} - R_{t,i,F} = \alpha_j + \beta_j MKT_{t,i} + \beta_{DII,j} DII_{t,i} + e_{t,i,j}, \tag{5}$$

where $R_{t,i,j}$ is the return on a security j, on day t for the intraday interval i, $R_{t,i,F}$ is the risk-free return that we approximate equal to 0, $DII_{t,i}$ is the C-Tz intraday diffusion index, $MKT_{t,i} = (R_{t,i,M} - R_{t,i,F})$ is the excess return on the Russell 3000 market portfolio and $e_{t,i,j}$ is a zero-mean residual. The use of high-frequency data allows us to obtain long samples of stock returns. Consequently, it is possible to run regressions using data from a reduced number of days and thus track how the significance of β_{DII} changes over time. We estimate the parameters of the model using non-overlapping rolling windows with a size of 22 days, which corresponds to 1,694 5-minute observations, or about one month of data. We observed high fractions of significant betas for almost all intervals from 2004 until 2015. This confirms that multivariate jumps help to explain stock returns by capturing common variations that are missed by the standard market factor and that might have some economic relevance when focusing on high-frequency data. Moreover, we do not observe higher levels of significance during the 2008 pre-crisis months but, instead, high picks clustered in 2007, 2008, 2010 and 2013.[5] Therefore, the DID performs well during calm periods, while the DII is more effective during more turbulent economic phases.

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[5] Tables and figures displaying such results, omitted here for the sake of space, are available upon request.

Table 2 $\hat{\beta}_{DID}$ **significance and DID** R^2_{adj} **variation.** Here, we compare the CAPM model defined in (5) with our two-factor model defined in (6). Regressions are subject to the condition that stocks presents at least 251 days (about a year) of non-null returns in the window of interest. Column $\hat{\beta}_{DID}$ reports the percentage of times in which $\hat{\beta}_{DID}$ in (6) is statistically significant at the 0.1 level in the window of interest. From the third to the seventh column we focus on the variation in the coefficient of determination (or CoD) we observe by moving from the CAPM to the two-factor model. Min and Max are respectively the minimum and the maximum difference values whereas $Q(0.25)$, Median and $Q(0.75)$ are the first, second and third quartiles of the same differences.

Window	$\hat{\beta}_{DID}$	R^2_{adj}(2-factor) $- R^2_{adj}$(CAPM)				
		Min	$Q(0.25)$	Median	$Q(0.75)$	Max
1998-2015	10%	-0.000	-0.000	-0.000	0.000	0.020
2002-2006	35%	-0.001	-0.001	-0.000	0.008	0.072
2007-2011	11%	-0.001	-0.001	-0.000	0.000	0.011
2012-2015	20%	-0.003	-0.001	-0.001	0.000	0.123

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