Space-time earthquake clustering: nearest-neighbor and stochastic declustering methods in comparison

Clustering spazio-temporale di terremoti: i metodi nearest-neighbor e di declustering stocastico a confronto

Elisa Varini, Antonella Peresan, Renata Rotondi, and Stefania Gentili

Abstract Earthquakes do not occur randomly in space and time; rather, they tend to group into clusters that can be classified according to their different properties, presumably related to the specific geophysical properties of a seismic region. Two methods for detection of earthquake clusters are considered in order to take advantage of different descriptions of the seismic process and assess consistency with the obtained clusters: the former is based on "nearest-neighbor distances" between events in space-time-energy domain; the latter is a stochastic method based on a branching point process, named Epidemic-Type Aftershock-Sequence (ETAS) model, which provides different plausible clustering scenarios by simulation. Both methods allow for a robust data-driven identification of seismic clusters, and permit to disclose possible complex features in the internal structure of the identified clusters. We aim at exploring the spatio-temporal features of earthquake clusters in Northeastern Italy, an area recently affected by low-to-moderate magnitude events, despite its high seismic hazard attested by historical destructive earthquakes.

Abstract *I terremoti non avvengono in modo casuale nello spazio-tempo, tendono piuttosto a raggrupparsi in cluster che possono essere classificati secondo le loro diverse proprietà, verosimilmente legate alle specifiche proprietà geofisiche della regione dove essi accadono. Due metodi per l'individuazione di cluster di terremoti sono stati presi in esame al fine di benificiare di descrizioni diverse del processo sismico e valutare la coerenza dei cluster ottenuti: il primo è un metodo basato sulla distanza "nearest-neighbor" tra coppie di eventi nel dominio spazio-tempo-energia; il secondo è un metodo stocastico basato su un modello di processi di punto di tipo branching noto come modello ETAS (Epidemic-Type Aftershock-Sequence)*

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che, attraverso tecniche di simulazione, è in grado di fornire diversi scenari plausibili di raggruppamento in clusters. Entrambe i metodi consentono una identificazione robusta dei cluster sismici, fondamentalmente guidata dai dati, e permettono di studiare la complessità nella struttura interna dei cluster stessi. Analizziamo le caratteristiche spazio-temporali dei cluster di terremoti avvenuti nell'Italia nord-orientale, un'area ad alta pericolosità sismica, come attestato da alcuni forti terremoti storici, e con una sismicità di magnitudo medio-bassa negli ultimi decenni.

Key words: earthquake clustering, simulation, stochastic declustering, ETAS model, nearest-neighbor distance

1 Some methods for earthquake clustering

Earthquake clustering is a prominent feature of seismic catalogs, both in time and space. Several methodologies for earthquake cluster identification have been proposed in the literature with at least a twofold scope: (1) characterization of the clustering features and their possible relation to physical properties of the crust; (2) declustering of earthquake catalogs which, by removing events temporally and spatially dependent on the mainshocks, allows for spatio-temporal analysis of the background (independent) seismicity. Nevertheless the application of different (de)clustering methods may lead to diverse classifications of earthquakes into main events and secondary events; consequently, the definition of mainshock is not univocal, but strictly related to the different physical/statistical assumptions underlying each method. Therefore we consider different declustering techniques to investigate classification similarities which might provide strong support for some clustering features, and classification differences which might highlight strength and lack of the clustering methods. Two clustering techniques are applied: the nearest-neighbor approach (Zaliapin and Ben-Zion, 2013) and the stochastic declustering approach (Zhuang et al., 2004). Both methods can be satisfactorily applied to decompose the seismic catalog into background seismicity and individual sequences (clusters) of earthquakes; moreover, they are data-driven and allow studying the internal structure of the clusters.

1.1 Nearest-neighbor method (NN)

Bak *et al.* [2] show that the waiting times between earthquakes in California follow a unified scaling law valid at different time scales (from tens of seconds to tens of years), by varying the magnitude threshold M and the linear size L of the studied area. The unified scaling law is obtained by rescaling the distribution $F(t)$ of the waiting times in such a way that t is replaced by $t10^{-bM}L^{d_f}$ and $F(t)$ by $t^{\alpha}F(t)$. Parameters α, b, and d_f correspond to empirical power laws which establish general relations among frequency, waiting times, magnitude, and epicentre locations of the earthquakes: α is the interval exponent of the Omori-Utsu law, b is the b-value of the Gutenberg-Richter distribution of the magnitude, and d_f is the spatial fractal dimension of the epicentre distribution. Thus the unified scaling law is a combination of scale invariant power functions that can be interpreted as statistical evidence of the self-organized critical behaviour of the earth dynamics, which results into a hierarchical organization of earthquakes in time, space, and magnitude. Based on this concept [1], the nearest-neighbor distance between two events in the space-time-energy domain is defined by:

$$\eta_{ij} = t_{ij}(r_{ij})^{d_f} 10^{-bM_i} , \tag{1}$$

where t_{ij} denotes the inter-occurrence time between events i and j ($i < j$), r_{ij} is the spatial distance between their epicentres, M_i is the magnitude of i-th event. Event i^* is the nearest-neighbor of event j if $i^* = argmin_{\{i:i<j\}} \eta_{ij}$; in other words, event j is an offspring of event i^*, and also i^* is the parent of j.

By connecting each event with its nearest-neighbor, one obtains a time-oriented tree where each event has a unique parent and may have multiple offspring. By setting a threshold distance η_0, a link between events i and j is removed if $\eta_{ij} > \eta_0$, for all i and j such that $i < j$. The removal of weak links leads to the identification of clusters of events; in each cluster, we define the largest magnitude event as mainshock, the events preceding the mainshock as foreshocks, and the events following the mainshock as aftershocks.

According to Zaliapin and Ben-Zion [6], the histogram of the distances between every pair of events clearly shows a bimodal distribution that can be approximated as a mixture of two Gaussian distributions, one associated with the Poissonian background activity (independent events) and the other with the clustered populations. Thus, threshold distance η_0 can be chosen as equal to the intersection point of the two estimated Gaussian distributions.

Parameters α, b, and d_f are estimated by the Unified Scaling Law for Earthquakes (USLE) method [3, 5].

1.2 Stochastic declustering method (SD)

The stochastic declustering approach is based on the space-time epidemic-type aftershock sequence (ETAS) model [4], a branching point process controlled by its intensity function $\lambda^*(t,x,y,M \mid \mathscr{H}_t)$ conditional on the observation history $\mathscr{H}_t$ up to time t.

Let (t_j, x_j, y_j, M_j) denote occurrence time, epicentral coordinates and magnitude of j-th event. Under the assumption of stationarity, ergodicity, and independence of the magnitude, the general expression of the conditional intensity function of ETAS model is given by:

$$\lambda^*(t,x,y,M \mid \mathscr{H}_t) = J(M)\lambda(t,x,y \mid \mathscr{H}_t) = \\ = J(M)\left[\mu(x,y) + \sum_{\{k:t_k<t\}} k(M_k)g(t-t_k)f(x-x_k,y-y_k \mid M_k)\right] ,$$

which, in the formulation of this study, decomposes into:

$$J(M) = be^{-b(M-M_c)} , \quad (2)$$

$$\mu(x,y) = \nu u(x,y) , \quad (3)$$

$$k(M) = Ae^{-\alpha(M-M_c)} , \quad (4)$$

$$g(t) = \begin{cases} (p-1)c^{p-1}(t+c)^{-p} & \text{for } t>0 \\ 0 & \text{otherwise} \end{cases} , \quad (5)$$

$$f(x,y \mid M) = \frac{(q-1)D^{2(q-1)}e^{\gamma(q-1)(M-M_0)}}{\pi[x^2+y^2+D^2e^{\gamma(M-M_0)}]^q} . \quad (6)$$

where M_c is the magnitude threshold of the catalog; Eq.(2) is the distribution of earthquake magnitude; the background rate in Eq.(3) is assumed to be constant in time; the expected number of events triggered from an event of magnitude M is expressed by Eq.(4); the probability density function of the occurrence times and the location distribution of the triggered events are given by Eqs.(5-6), respectively. It is also worth defining the total spatial intensity $m(x,y) = \lim_{T\to\infty}\int_0^T \lambda(t,x,y \mid \mathscr{H}_t)dt/T$ and the clustering spatial intensity $\gamma(x,y) = m(x,y) - \mu(x,y)$. The model parameters are denoted by ν, A, c, α, p, D, q, and γ.

In ETAS model, background earthquakes independently occur at a Poisson rate constant in time, triggering other events with a spatio-temporal decay modelled by the Omori-Utsu law; triggered events have, in turn, the ability to trigger other events. The following expressions respectively provide the probability that event j is triggered by previous event i, the probability that event j is triggered in general, and the probability that event j is generated by the background process:

$$\rho_{ij} = \frac{k(M_i)g(t_j-t_i)f(x_j-x_i,y_j-y_i \mid M_i)}{\lambda(t_j,x_j,y_j \mid \mathscr{H}_{t_j})} \quad (7)$$

$$\rho_j = \sum_{i:t_i<t_j} \rho_{ij} \quad (8)$$

$$\varphi_j = 1-\rho_j \quad (9)$$

By simulating according to these probabilities, the dataset splits into two subsets which are realizations of the background process and the triggered process, respectively: the former is the declustered catalog and the latter identifies a set of clusters each starting from a background event.

The estimation of ETAS parameters is performed by an iterative algorithm that simultaneously estimates the background rate by a variable kernel method and the model parameters by the maximum likelihood method; then the branching structure is obtained by simulation [7, 8, 9].

2 Case study: Northeastern Italy seismicity

The comparative analysis of earthquake clusters is carried out for a sequence of earthquakes occurred in Northeastern Italy. In this area only low-to-moderate magnitude events have been recorded during the last decades, despite its high seismic hazard attested by at least eight historical destructive earthquakes occurred since 1348, the most recent one being the 1976 May 6 M6.4 earthquake, located in the Julian Prealps. A further aim of the clustering analysis is to provide a quantitative basis to understand the role of moderate size earthquakes in the framework of regional seismicity.

2.1 Data

The Italian National Institute of Oceanography and Experimental Geophysics (OGS) started with the monitoring of seismic activities in the Northeastern area of Italy since 1977. We consider the set of earthquakes reported in the OGS bulletins, occurred from 1977 to 2015, with local magnitude at least $M_c = 2$, and ranging from 11.5°E to 14.0°E in longitude and from 45.5°N to 47.0°N in latitude (Fig. 1).
The dataset is considered as complete except for the early 1990s, when data are missing due to a fire accident [5].

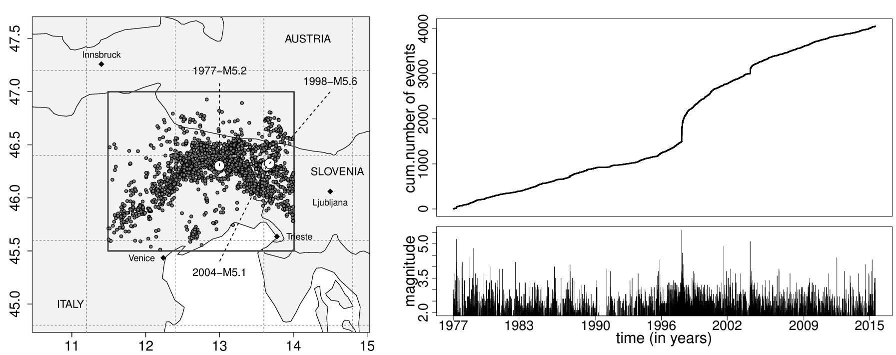

Fig. 1 Epicentres of the earthquakes in the study area: black point if magnitude $M \leq 5$, white points otherwise [left]. Cumulative number of earthquakes versus time (top) and magnitude versus time (bottom) [right].

2.2 Results

NN method univocally splits the dataset into two subsets, a set of background events and a set of triggered events; more details are given in [5]. A tree representation of the identified clusters is provided in order to better highlight the complexity of clusters structure. For example Fig. 2(a) shows the tree of the cluster related to the 1998 earthquake, with magnitude $M5.6$.

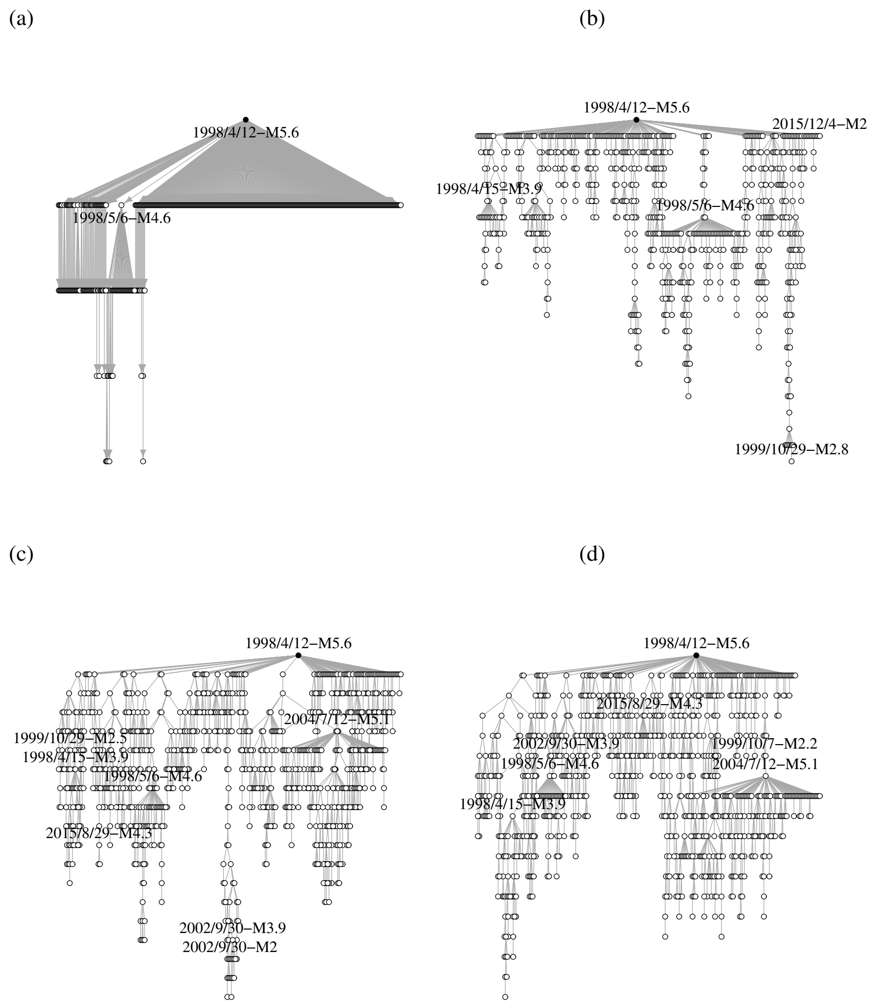

Fig. 2 Cluster trees of the 1998/4/12 earthquake, magnitude M5.6: (a) NN method, (b) SD method, the most probable scenario, and (c-d) SD method, two simulated scenarios, respectively.

As for the SD method, each event j in the dataset is associated with the estimated probability $\hat{\varphi}_j$ ($\hat{\rho}_j$) to be a background (triggered) event. These probabilities provide different clustering scenarios: Fig. 2(b) shows the cluster tree of the 1998 earthquake where each $j-$th event is associated with the most probable ancestor i^* ($i^* = argmax_{i<j}\rho_{ij}$), while Fig. 2(c-d) show the cluster trees of the same earthquake, each derived by simulation according to the estimated background probabilities. For

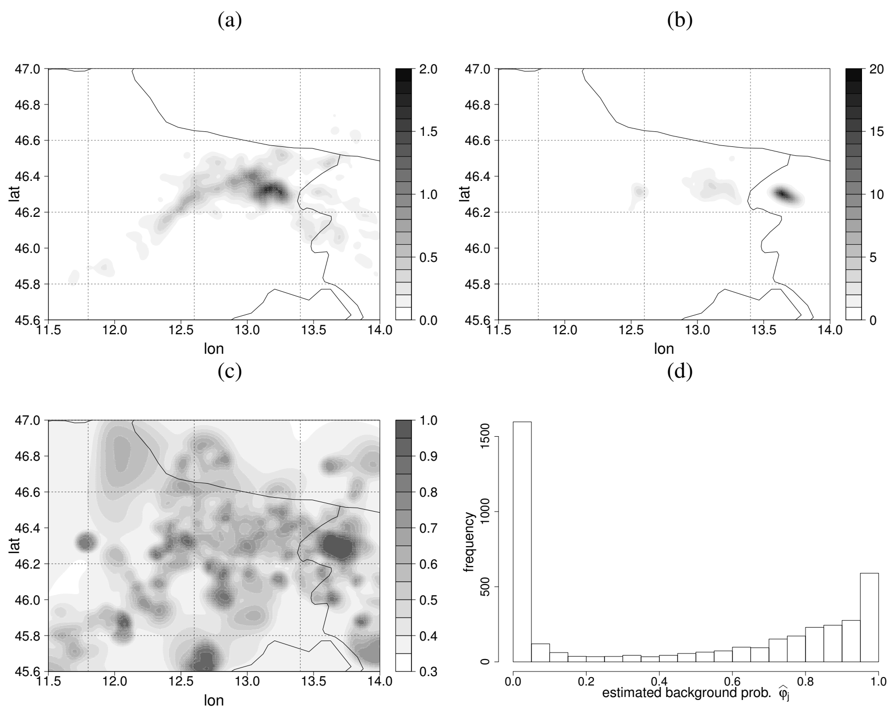

Fig. 3 Estimated background rates (a), estimated clustering rates (b), ratio between estimated clustering rate and estimated total rate (c), and histogram of the estimated background probabilities (d).

all the spatial coordinates (x,y) in the study area, with reference to SD method, Fig. 3 shows: (a) the estimated background rates $\hat{\mu}(x,y)$, (b) the estimated clustering rate $\hat{\gamma}(x,y)$, (c) the ratio between between estimated clustering rate $\hat{\gamma}(x,y)$ and estimated total rate $\hat{m}(x,y)$, which can be regarded as a smoothed approximation of the triggering probabilities ρ_j. Fig. 3(d) shows the histogram of the estimated background probabilities $\hat{\varphi}_j$, from which the triggering probabilities are given by $\hat{\rho}_j = 1 - \hat{\varphi}_j$: most of the events have high probability of being either background events (21% for $\hat{\varphi}_j \geq 0.9$) or triggered events (42% for $\hat{\varphi}_j \leq 0.1$), and the remaining events have less decisive probabilities (about 37% of the data have background probability ranging from 0.1 to 0.9).

A preliminary comparison of results from the two methods shows that the cluster structures produced by NN and SD approaches have comparable trend in terms of spatial extent of seismic clusters. But SD method tends to find some connections between events close in space even if far in time. With reference to 1998 earthquake (Fig. 2), its NN-cluster includes only 480 earthquakes while SD-clusters have more than 900 earthquakes, on average; this is due to the presence in the SD-clusters of earthquakes occurred years later (e.g. in 2002, 2004, or even 2015).

For large sequences, trees obtained from the SD method show a more complex internal structure than trees obtained by the NN method. To quantify topological differences among trees, the average node depth and average leaf depth are considered. The former is the average number of links that connects each node of the tree root; the latter is similarly defined as the average number of links that connects each leaf (node without descendant) to the tree root. According to these scalar measures of internal complexity of a tree, average node depth and average leaf depth are, respectively, 1.30 and 1.31 for the NN-cluster in Fig. 2(a), 5.59 and 5.59 for the SD-cluster in Fig. 2(b), 5.90 and 6.14 for the SD-cluster in Fig. 2(c), 6.20 and 6.43 for the SD-cluster in Fig. 2(d). Greater complexity of the clusters identified by SD method reflects the multilevel triggering property of the ETAS model; we recall that ETAS model assumes that each event is able to generate offspring.

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