Spatiotemporal Prevision for Emergency Medical System Events in Milan

Andrea Gilardi, Riccardo Borgoni, Andrea Pagliosa, Rodolfo Bonora

Abstract Organizing the emergency medical system in a big city is an extremely difficult task given the huge number of people that everyday pass through the city area. In this paper we employ a spatio-temporal process to model the emergency event occurrences in Milan. The proposed approach has been found effective in predicting events through the city area and computationally efficient despite the big amount of data to be processed.
Abstract *L'organizzazione di un servizio di emergenza sul territorio risulta essere un compito complesso nelle aree metropolitane come Milano dato l'enorme numero di persone che vi transitano quotidianamente. In questo lavoro si adotta un modello spazio temporale per rappresentare la dinamica delle chiamate di emergenza sul territorio del capologuo Lombardo. Il metodo adottato si é dimostrato essere efficace nel prevedere gli eventi sul territorio comunale nel periodo di tempo considerato e computazionalmente efficiente nonostante la consistente mole di dati da elaborare.*

Key words: emergency medical system, spatio-temporal point process

1 Introduction

Given the large number of people that everyday pass through the metropolitan area, the organization of the emergency medical system (EMS) in Milan is an extremely

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difficult task. Numerous studies have proposed different models for the optimal allocation of ambulances in the territory and each of these models is based on ad-hoc predictions for the future locations of emergency events. In this paper we implemented an algorithm that predicts the distribution of the ambulance interventions in Milan for every hour of the day and every place of the urban area.

All ambulance dispatches from 1st of January 2015 till 25th of September 2017 have been considered for a total of, approximately, 500'000 events. The spatial distribution of the events is reported in (Figure 1-(a)). The figure clearly points out an anomalous pick of event intensity in the north-west part of the map caused by the universal exposition hosted by Milan (EXPO) that took place in this area in 2015. All the events occurred in the EXPO area have been removed from the subsequent analysis.

This type of data is challenging for several reasons:

Sparsity: even if the dataset is extremely large, there are only 21 events per hour on average scattered on an area of about 180 km^2;

Computational challenges: the numerical estimation of a spatio-temporal model is particularly difficult considering the long training time of the algorithms;

Seasonality: the total number of events per hour exhibits both daily and weakly seasonality (Figure 1-(b)).

2 Spatio-Temporal Model

Using the approach suggested in [5], we modeled Milano's ambulance demand on a continuous spatial domain $S \in \mathbb{R}^2$ and a discretized temporal domain of one-hour intervals $T = \{1, 2, \dots, \}$. We assumed that ambulance demand follows a Non-homogeneous Poisson Process [1] with intensity function $\lambda_t(\mathbf{s})$ for each time period t. Furthermore, we decomposed this intensity function as

$$\lambda_t(\mathbf{s}) = \delta_t f_t(\mathbf{s}), \quad \mathbf{s} \in S \subseteq \mathbb{R}^2,\ t \in \mathbb{N} \tag{1}$$

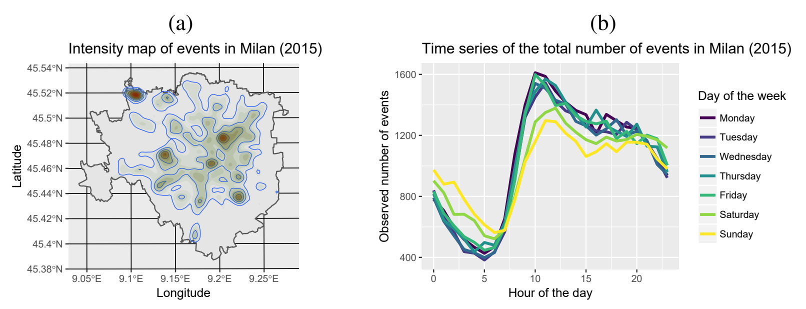

Fig. 1 Spatial distribution of the events in Milan during 2015 (a). Temporal distribution of the number of interventions per hour of the day (b).

where δ_t models the expected number of events during the period t in the region S and $f_t(\mathbf{s})$ represents the continuous spatial density of the ambulance demand at time t. A dynamic latent factor structure has been assumed for the temporal component and it has been estimated iteratively (Section 2.1). The spatial component has been estimated non-parametrically via a weighted kernel (Section 2.2).

2.1 The temporal component

Following the approach suggested in [3, 2], we assumed a dynamic latent factor model for the temporal component δ_t and estimated it using smoothing splines [4]. More specifically we supposed that it is possible to predict the mean value δ_t using a set of deterministic covariates, namely the hour of the day, the day of the week and the week of the year, that were included in the model by applying constraints on the factor loadings as explained below.

To avoid negative values we modelled the intra-day pattern on the log scale and we assumed that it can be approximated using a linear combination of a small number K of factors[1] i.e.

$$\log \delta_{ij} = L_{i1} f_{1j} + \ldots + L_{iK} f_{Kj}, \quad i = 1, \ldots, 365, \quad j = 1, \ldots, 24. \tag{2}$$

The model can be expressed in matrix form as $\log \Delta = LF'$. The matrix L is further partitioned as

$$\log \Delta = LF' = (H_1 B_1 + H_2 B_2) F' \tag{3}$$

where H_1 and H_2 are the incidence matrix that identifies the day of the week and the incidence matrix that identifies the week of the year respectively, whereas B_1 and B_2 are matrices of unknown coefficients with suitable dimensions.

Since both F and L are unknown matrices, we implemented the following algorithm [2] to estimate them:

1. using singular value decomposition we decomposed the matrix of the observed counts logarithms as UDV' and we obtained an initial estimate $L = UD$ and $F' = V'$;
2. considering F as known, we updated the estimate of L using a *gam* model with a Poisson response variable and the day of the week and the week of the year as covariates;
3. considering L as known, we updated the estimate of F using again a *gam* model with a Poisson response and the hour of the day as covariate;
4. we iterated 2 and 3 until convergence.

[1] The value of K was chosen by training the model on the data collected in 2015 data validating it using 2016 data.

2.2 The spatial component

The continuous spatial density function $f(\cdot)$ has been estimated at a future time period u using the spatio-temporal weighted kernel density estimator [5]

$$f_u(\mathbf{s}) = \frac{\sum_{t \in \mathscr{T}_{obs}} \omega(\mathbf{s}_{i,t}, u) K_{\mathbf{H}}(\mathbf{s} - \mathbf{s}_{t,i})}{\sum_{t \in \mathscr{T}_{obs}} \omega(\mathbf{s}_{i,t}, u)}, \qquad \mathbf{s} \in S \tag{4}$$

where $\mathscr{T}_{obs}$ represents a set of past periods, $\omega(\cdot)$ a weight function and $K_{\mathbf{H}}(\cdot)$ a multivariate gaussian Kernel with bandwidth matrix $\mathbf{H}$.

Figure 1-(b) shows that the observed number of events exhibits both daily and weakly seasonality. The aim of the weight function is to take advantage of these patterns and select those observations that are mostly influential in predicting the density function at a generic future time point u. Analysing the data it emerged that the strength of these patterns was different in different areas of Milan. For this reason we divided the municipality in $C = 9$ neighbourhoods and the weight function was estimated separately in each cell.

We assumed [5] that the prediction at a future time point u depends only upon the temporal lag between u and the event at time t. The impact of the event occurred at time t is weighted by the following weight function that measures how two observations located in a cell c are positively correlated:

$$\omega_c(u-t) = \rho_{1,c}^{u-t} + \rho_{2,c}^{u-t} \rho_{3,c}^{\sin^2\left(\frac{\pi(u-t)}{24}\right)} \rho_{4,c}^{\sin^2\left(\frac{\pi(u-t)}{168}\right)}, \quad c \in \{1, \ldots, C\} \tag{5}$$

where ρ_1 describes any short-term seasonality, ρ_3 and ρ_4 express daily and weakly seasonality respectively and ρ_2 is a discount factor. Since using a likelihood approach to estimate these coefficients is prohibitive due to the computational costs, we implemented the algorithm suggested in [5]. Hence $\rho_1, \ldots, \rho_4$ have been estimated by minimizing the following quantity:

$$\min_{\rho_{j,c}, j \in \{0,\ldots,4,\}} \sum_{l=1}^{M} (A_c^+(l) - \rho_{0,c} \omega_c(l))^2 \tag{6}$$

where A_c^+ represents the positive part of the autocorrelation function of the proportion of events in the spatial cell c, M is the maximum lag considered in the autocorrelation function and $\rho_{0,c}$ is a normalizing constant. The minimization problem was worked out using the *optim* function of R [6].

3 Results and conclusions

We trained the algorithm described in section 2.1 using the data from 2015. The number of factors K was identified via an external cross validation based on the dataset collected in 2016. $K = 4$ is the optimal value suggested by the procedure. Finally the model has been tested using again the 2016 data. The results are displayed in Figure 2 where the scatter plot of the predicted counts versus the observed counts is reported. A boxplot of predictions is drawn at each observed frequency to improve the graph readability. Patterns somehow similar were found for the two years. Despite a great variability and some potential outlying values, the predictions seem to replicate reasonably well the observed data both for the train set (year 2015) and for the test set (year 2016).

The spatial dynamic has been estimated using the procedure detailed in Section 2.2. First we solved the minimization problem mentioned above to obtain an estimate of the coefficients for the weight function. Then we estimated the spatial density using Equation 4 for one particular day. To exemplify the procedure we considered two maps for the 26th of September 2017, one estimated at 02:00 AM and the other estimated at 2:00 PM. The results are displayed in Figure 3 where darker areas are associated to higher values of the spatial density. There are clear differences between the two plots: during daytime hours the events are mostly concentrated around popular places and crossing points such as the Duomo area or the Central Station areas whereas during the night interventions are more scattered over the whole territory of the municipality.

This study demonstrates how resorting to a spatio-temporal non homogeneous Poisson model is adequate to represent the temporal and the spatial patterns that are present in the EMS data of Milan. Some areas and some hours of the day are found more critical for emergency events and this is a preliminary step to support local authorities in optimally allocating ambulances and resources in the territory. From the statistical modelling perspective some further enhancements can be introduced to improve the diagnostic, for instance developing algorithms to simulate events from the estimated model and to create measures of the prediction performance. Moreover, alternative specifications for the weight function should be also considered. Finally, a comparison between the temporal and spatial model suggested in this paper and other approaches such as machine learning algorithms (for instance, boosting or random forest) can be also useful to identify the best modelling strategy to support authorities in the day by day activity.

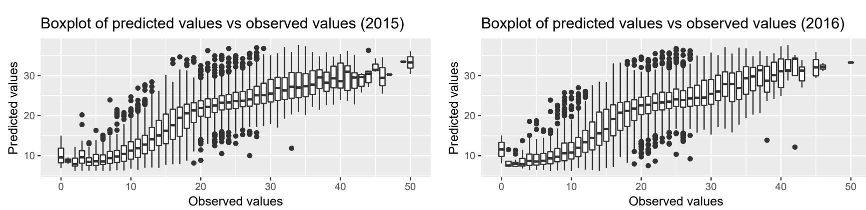

Fig. 2 In these two plots we display the predictive performance of the dynamic latent factor model for the training data (2015, left plot) and for the test data (2016, right plot). We can see that, despite high variance in the estimates, both plots show good predictive performances.

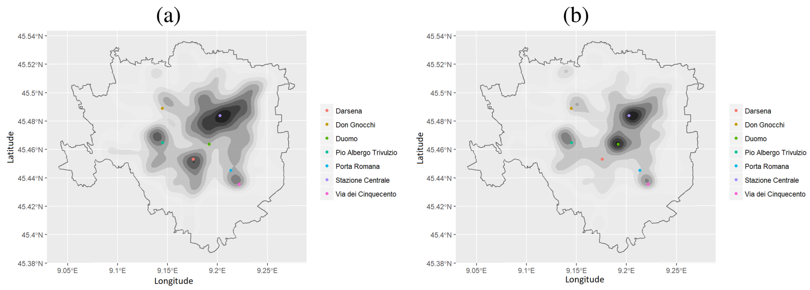

Fig. 3 Spatial density estimate for the 26th of September 2017 at 02:00 AM (a) and 02:00 PM (b). Some important places of the city are highlighted on both maps. Darker areas are associated to higher values of the spatial density.

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