Design-based maps for two-phase environmental surveys

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Environmental surveys generally address

- design-based estimation of descriptive summaries, such as forest cover and total biomass
- conservative estimation of the design-based variances
- mapping for a visual overview of the spatial pattern of an attribute of interest, usually performed in a model-based approach

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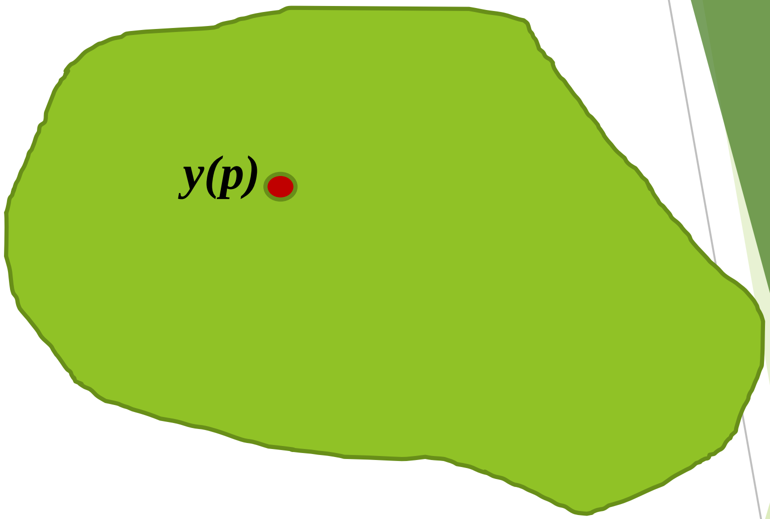
In a **design-based framework**

Some general notation and statement of the problem

$\mathcal{A}$ study region of size A
(connected and compact set of $\mathbb{R}^2$)

$p \in \mathcal{A}$ population unit

$y(p)$ amount of the survey variable
Y at $p \in \mathcal{A}$



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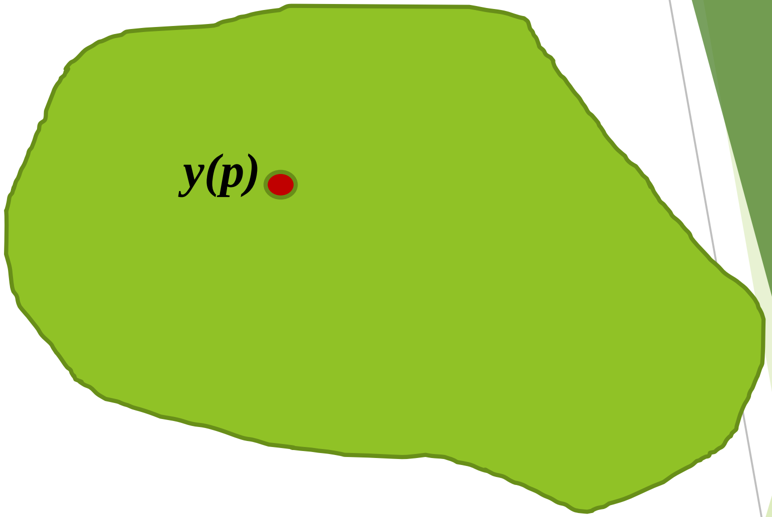

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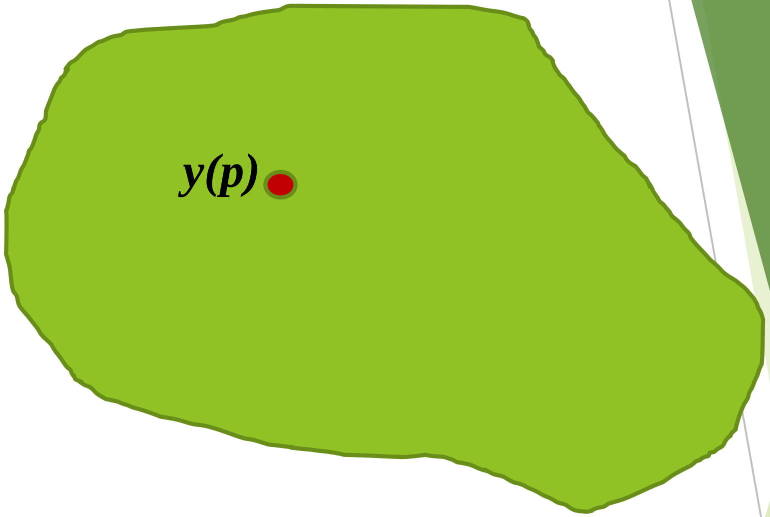

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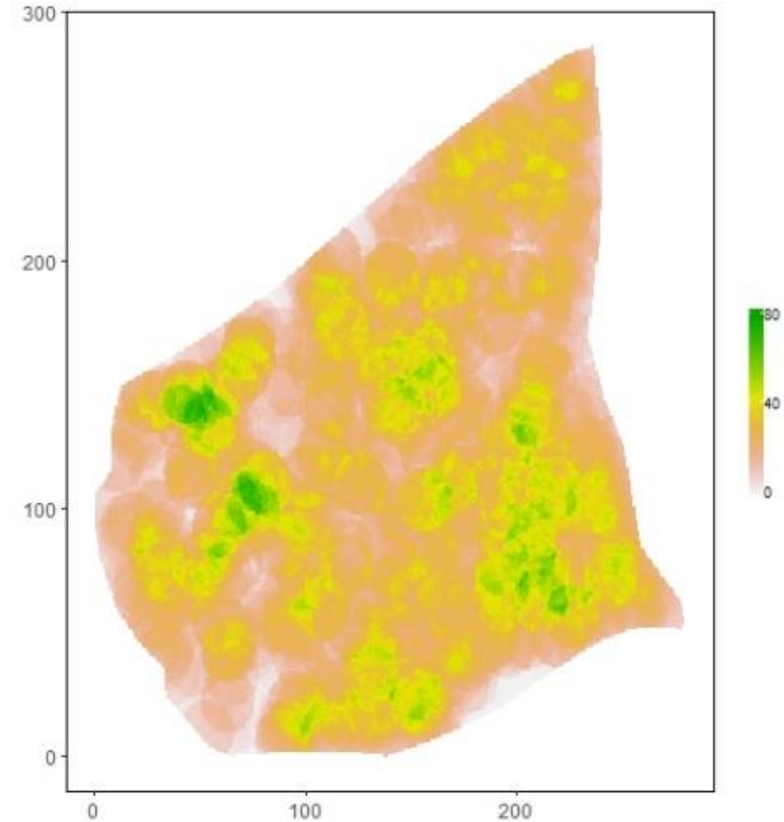
Aim:
to obtain a whole map of the population,
i.e. estimating $y(p)$ for any $p \in \mathcal{A}$
on the basis of a sample of points selected from $\mathcal{A}$

Example

$y(p)$ is the basal area (m^2) within a circular plot of radius $13m$ in Val di Sella (northeastern Italy, southern Alpine area)



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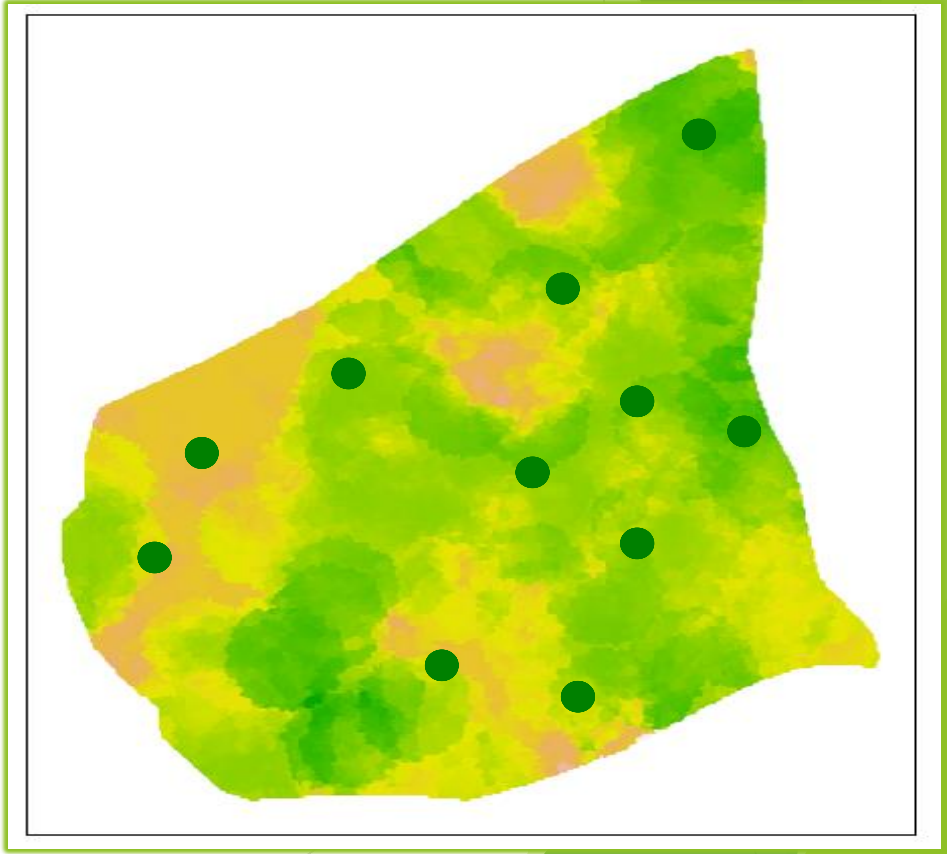

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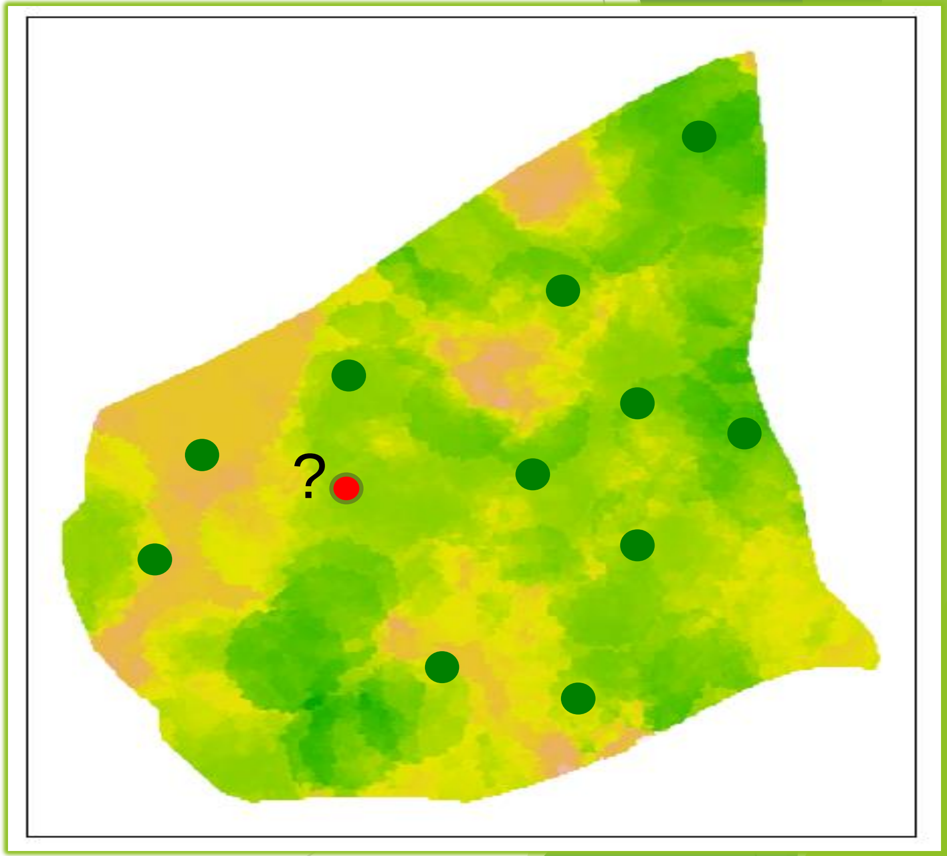
a criterion is adopted to estimate $y(p)$ at an unsampled location p



Model-dependent inference

under model-dependent approaches the properties of the resulting maps are determined by the

super-population (model)

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The most common techniques are

- the kriging predictors (e.g. Cressie 1993)
- other model-dependent methods exploiting the auxiliary information
  - regression kriging (Christensen, 2001)
  - cokriging (Cressie 1993)
  - locally weighted regression (Cleveland and Devlin 1988)
  - *k*-nearest neighbour (McRoberts et al. 2007)

Design-based framework

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drawbacks and merits of design-based and model-based approaches are well delineated in statistical literature:

e.g. Smith 2001, Gregoire 1998, Thompson 2002,
Schreuder et al. 1993

"Design-based inference is objective, nobody can challenge that the sample was really selected according to the given sampling design. The probability distribution associated with the design is real, not modelled or assumed" Särndal et al. (1992, p. 21)

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any second-phase point is visited
and survey variable is recorded

First-phase estimation

(Fattorini et al. 2017, Biometrika)

If all the first-phase n points $P_1, P_2, \ldots, P_n$ were visited on the ground

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$y(P_1)$, $y(P_2), \ldots,$ $y(P_n)$ were recorded

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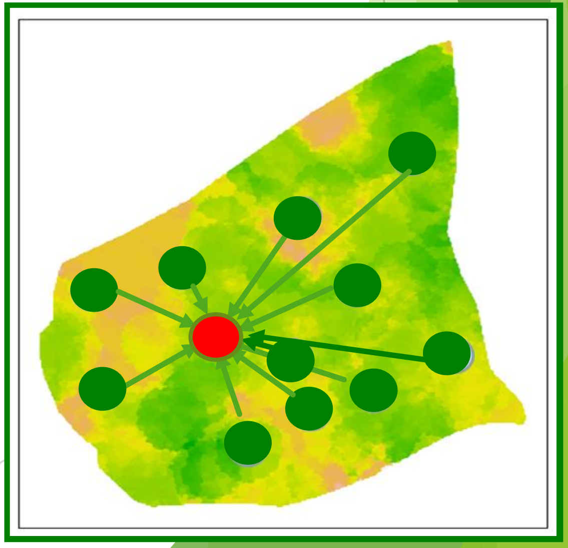
$$\hat{y}(p) = \sum_{i=1}^{n} w_i(p, P_i)\, y(P_i)$$



sampled first-phase points

unsampled location

$\mathrm{E}\left(\hat{y}(p)\right) \neq y(p)$ Bias how much?

$\mathrm{V}\left(\hat{y}(p)\right) = ?$ Precision?

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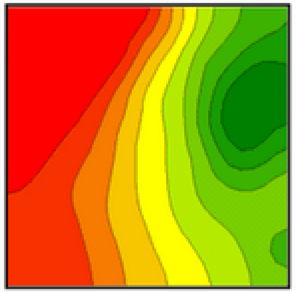
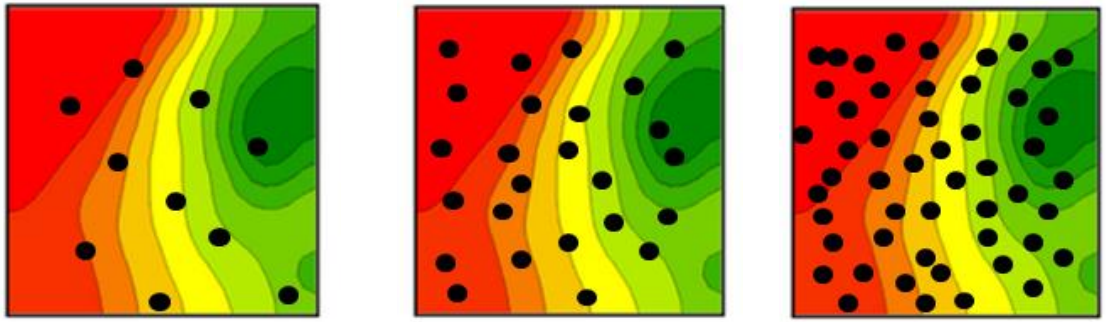
Investigations are needed
for DBAU&C

Asymptotic framework



The surface *y*(*p*) is fixed

Suppose a sequence of designs $\{D_k\}$ to select a sample of locations $S_k = (P_1^{(k)}, \ldots, P_{n_k}^{(k)})$ of size n_k from $\mathcal{A}$, with $n_k \to \infty$ as *k* increases



…

Properties of the first-phase IDW interpolator

Under suitable assumptions they proved that, as the number of sampled locations increases, the estimated map converges to the true map unless a set of zero measure, that is

$$\lim_{n\to\infty} \int_{\mathcal{A}} \mathrm{E}\{|\hat{y}(p) - y(p)|\}\, dp = 0$$

Fattorini et al. (2017)

The IDW interpolator is almost everywhere **DAU&C**

- **D**esign **A**symptotically **U**nbiased
- **C**onsistent

Conditions for DAU&C of the first-phase IDW interpolator

C1. $y(p)$ is a piecewise continuous function

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In real world there are parts of the study region in which density changes smoothly throughout space, well approaching the theoretical condition of continuity

Even when density changes abruptly, that usually occurs along borders delineating sudden variations in the characteristic of the study region

These borders may be realistically approximated by curves well approaching the theoretical condition of discontinuity over a region of zero measure

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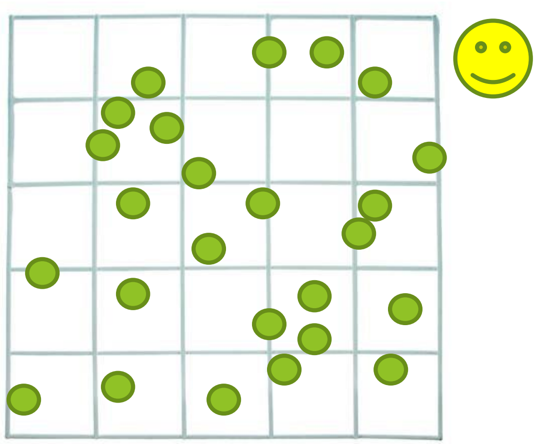
C1. $y(p)$ is a piecewise continuous function

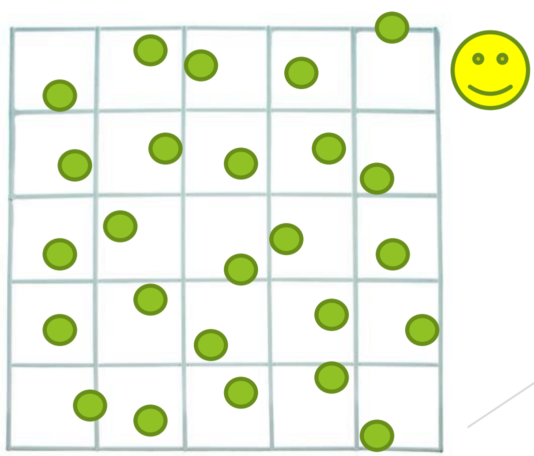
C2. weights should decrease as a power of distance

C3. for large n the sampling scheme is able to evenly spread the sample points in such a way that any unsampled location is likely to have neighbouring points sampled → asymptotical spatial balance

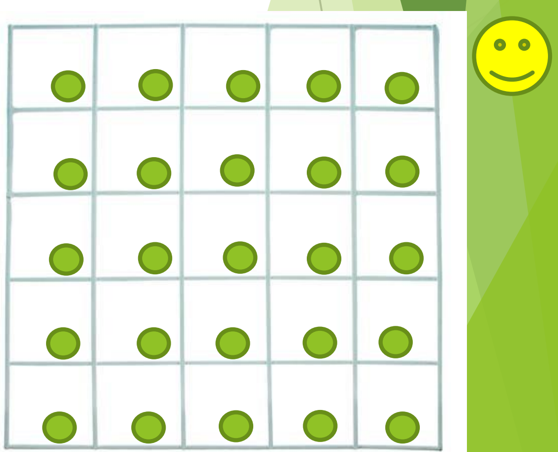
Uniform random sampling



Tesselation stratified sampling



Systematic grid sampling



What about DAU&C
when a two-phase sampling
is implemented?

Second-phase estimation

(work in progress)

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- a sample S of m first-phase points is selected by means of a suitable sampling scheme inducing first-order inclusion probabilities π_i , $i \in S$
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Estimation of values at the unsampled locations
can be performed by
two-phase IDW interpolators

$$\hat{y}(p, S) = \frac{\sum_{i=1}^{n} w_i(p, P_i) y(P_i) G_i}{\sum_{i=1}^{n} w_i(p, P_i) G_i}$$

where G_i are not-negative random variables equal to 0
if point i has not been selected in the second-phase sample S

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$$\boxed{\text{different choices of } G_i\text{s give rise to different estimators}}$$

Two-phase ratio and direct interpolators

- Two-phase ratio interpolator: $G_i = 1/\pi_i \quad i \in S$

$$\hat{y}_R(p, S) = \frac{\sum_{i \in S} w_i(p, P_i) y(P_i) \pi_i^{-1}}{\sum_{i \in S} w_i(p, P_i)\, \pi_i^{-1}}$$

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- Two-phase direct interpolator : $G_i = 1 \quad i \in S$

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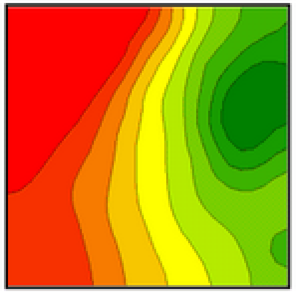
When $\pi_i = \pi_o$ for any $i \in S$ $\quad \hat{y}_R(p,S) = \hat{y}_D(p,S)$

Asymptotic framework



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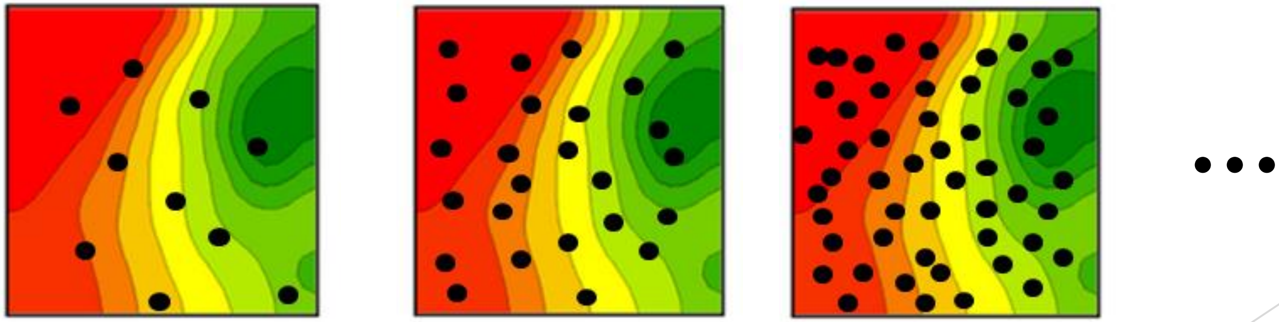
Suppose a sequence of designs $\{D_k\}$ to select a second phase sample of locations $S_k = (P_1^{(k)}, \ldots, P_{m_k}^{(k)})$ of size m_k from $\mathcal{A}$, $n_k \to \infty$, $m_k \to \infty$ as k increases



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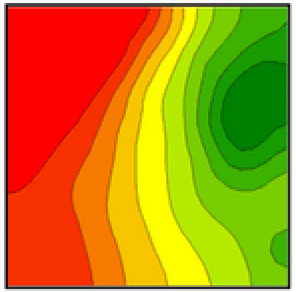

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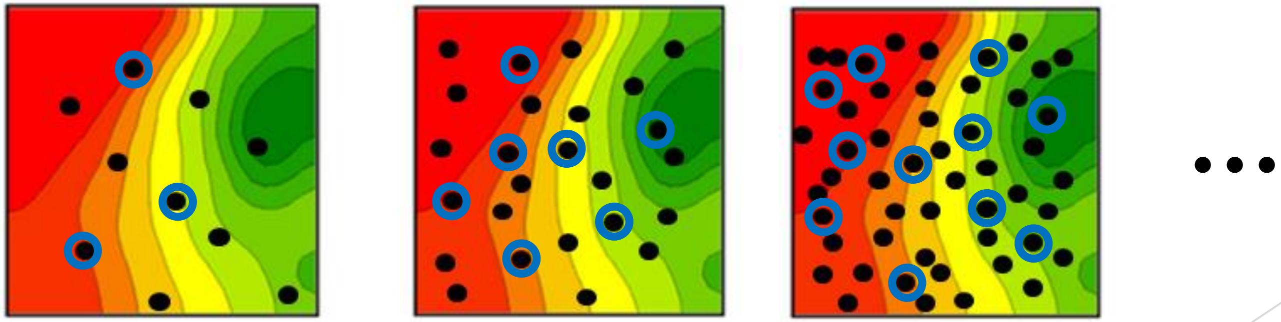

Asymptotic framework

Sequence of IDW interpolators

$$\hat{y}_k(p,S) = \frac{\sum_{i=1}^{n} w_i\left(p, P_i^{(k)}\right) y(P_i^{(k)}) G_i}{\sum_{i=1}^{n} w_i\left(p, P_i^{(k)}\right) G_i}$$

The IDW interpolator is defined to be **point-wise design-consistent** at $p \in \mathcal{A}$ if

$$p\lim_{k\to\infty} \left|\hat{y}_k(p) - y(p)\right| = 0$$

Properties of the two-phase interpolators

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D3. for any $p \in \mathcal{A}$ and any $\epsilon > 0$ there exists $T > 0$ and an integer k_0 such that

$$\Pr\left\{Z_k(p, T/\sqrt{m_k}, \mathcal{S}_k) = 0\right\} < \varepsilon \quad \forall\, k > k_0$$

D4. $|G_i| \leq const$

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the two-phase IDW interpolators
are almost everywhere **DAU&C**

Some two-phase sampling schemes ensuring DAU&C of the two-phase IDW interpolators

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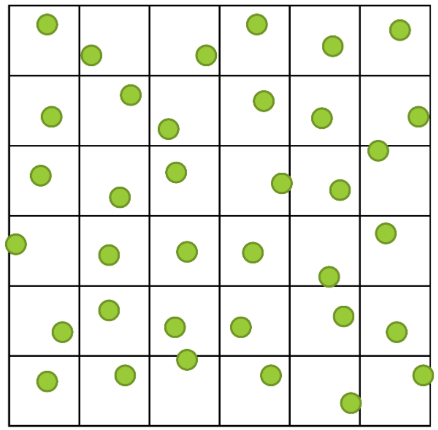
First phase:
Tesselation stratified sampling



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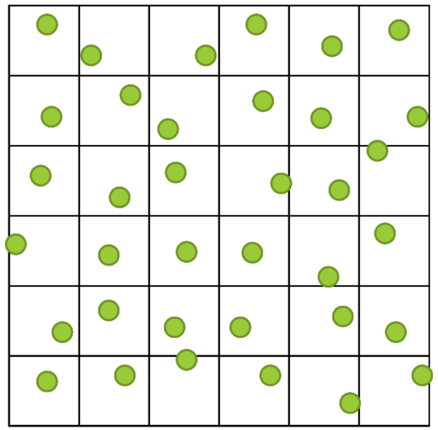
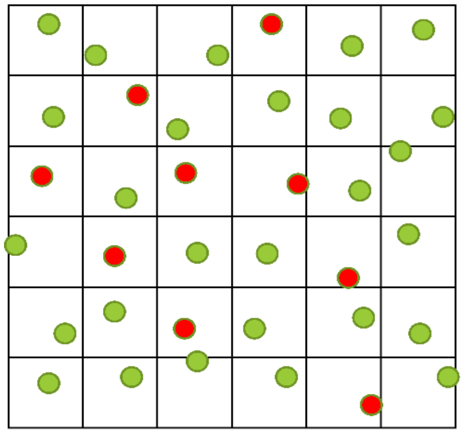


Second phase:
Simple random sampling without replacement



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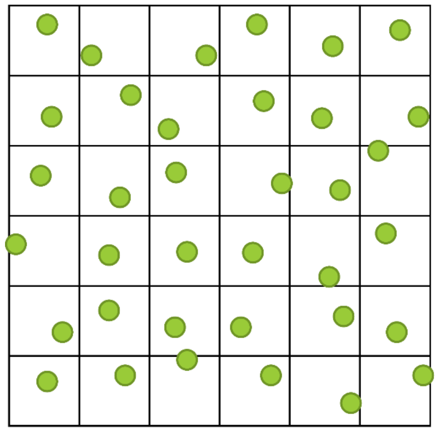

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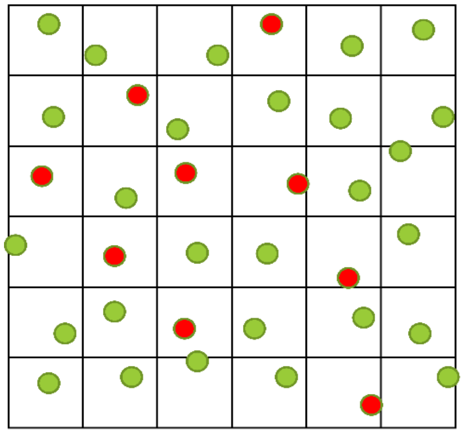

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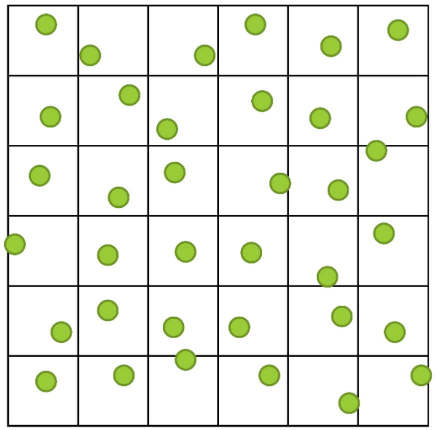

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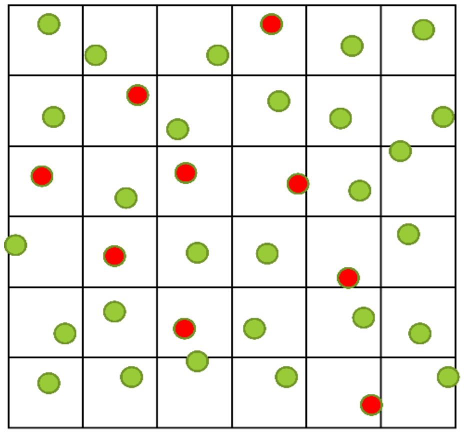

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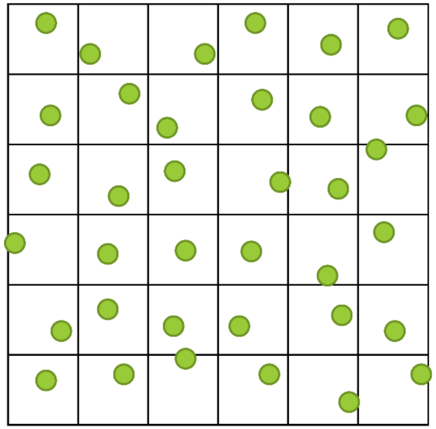

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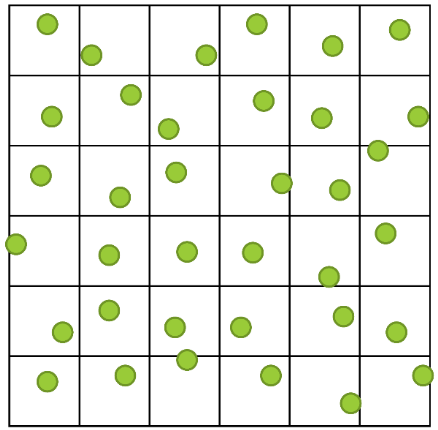

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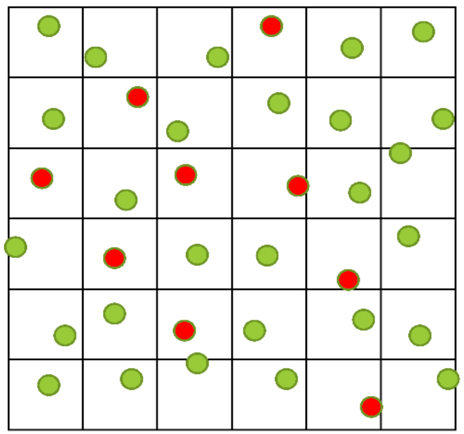

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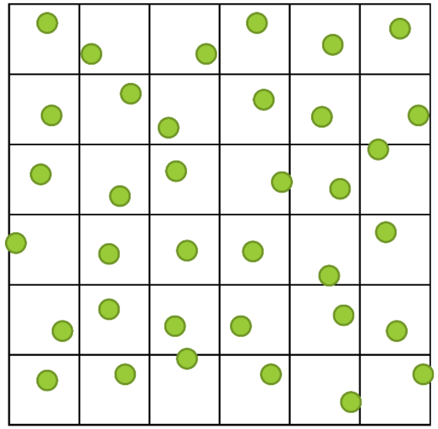
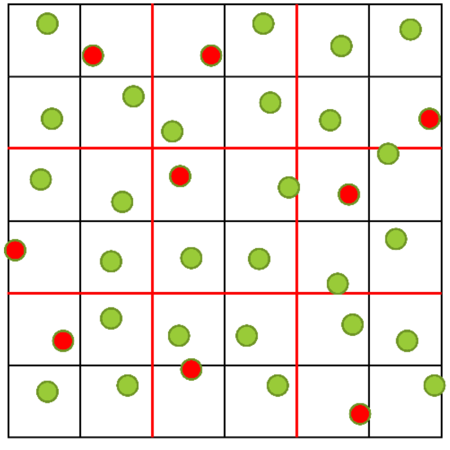


Second phase:
One-per-stratum-stratified sampling



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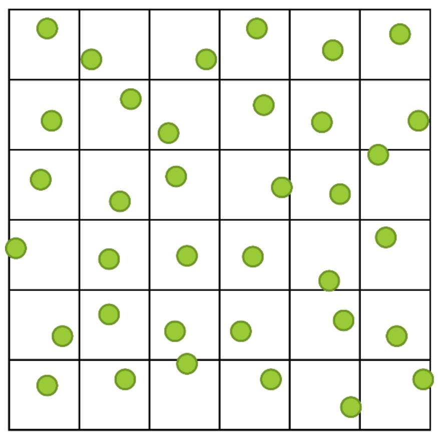

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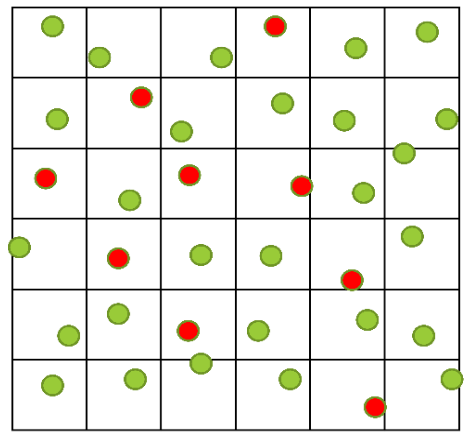

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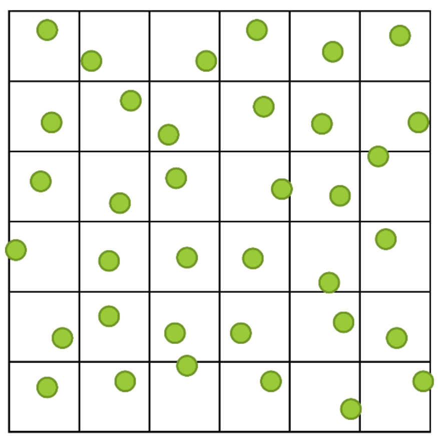

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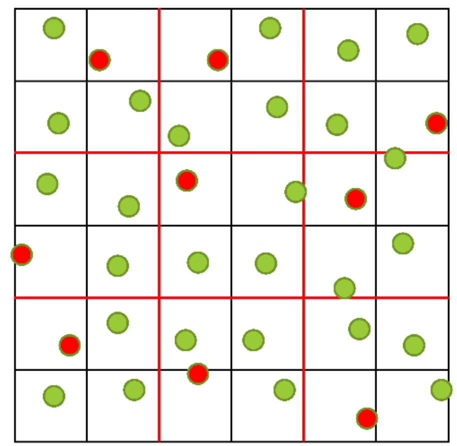

Mean square estimator

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- B bootstrap samples S_b (b=1,...,B) are selected from the map following the original sampling scheme

Mean square estimator

The pseudo-population bootstrap method is used

- the pseudo-population is the map obtained by means of the two-phase IDW interpolator
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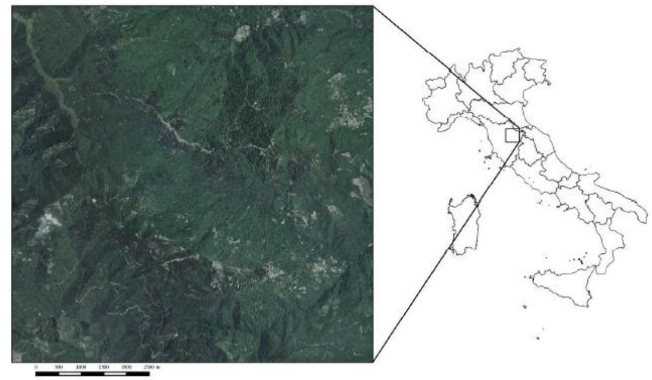
- $\hat{y}_b^*(p, S_b)$ is the two-phase interpolator with the b-th bootstrap sample
- the mean square error of $\hat{y}(p, S)$ can be estimated by

$$\frac{1}{B}\sum_{b=1}^{B}(\hat{y}_b^*(p, S_b) - \hat{y}(p, S))^2$$

Case study

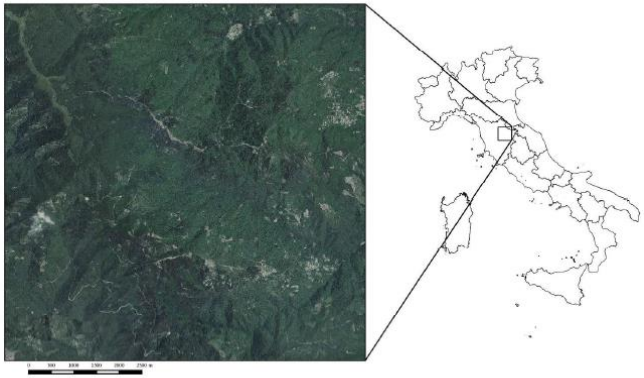
Case study



Study area

- size 4,900 ha
- located in the Casentino Valley, in Central Italy
- prevailing tree species are beech, European black pine, Turkey oak, Douglas fir, and silver fir.

Case study



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Attribute of interest

$y(p)$ total volume of all trees - taller than 1.30 *m* and with stem diameter at breast height greater than 5 *cm* - lying within the circular plot of radius 20 *m* centred at p

Case study

SAMPLING: two-phase sampling

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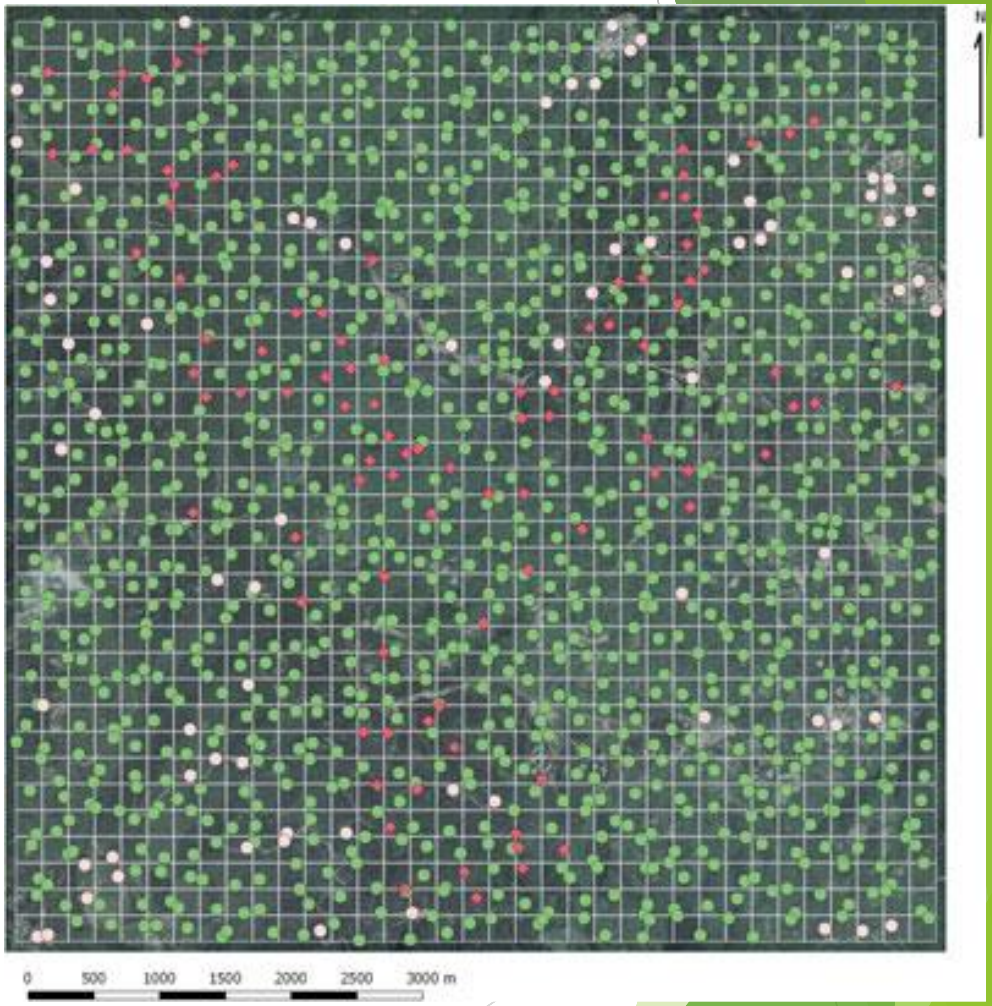
  - each second phase points was visited, recording the total volume of all the trees lying within the circular plot of radius 20 *m* centered at the point

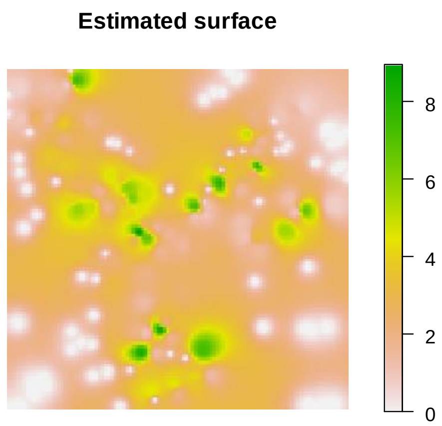
Case study

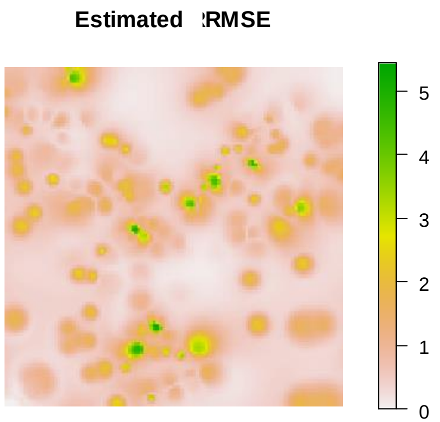
- 74 first-phase plots located outside forest
- 1056 first-phase plots located within forest and not visited on the ground
- 95 second-phase plots visited on the ground



Case study

Two-phase ratio interpolator





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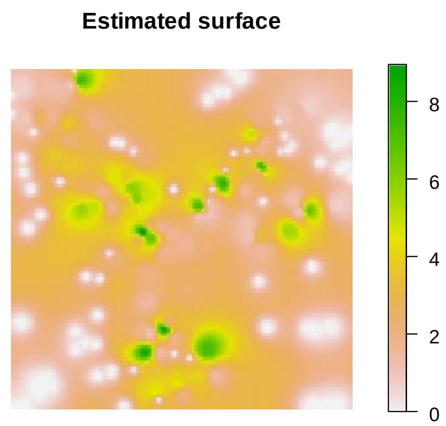
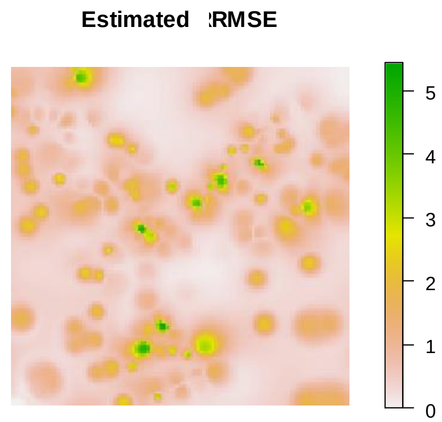
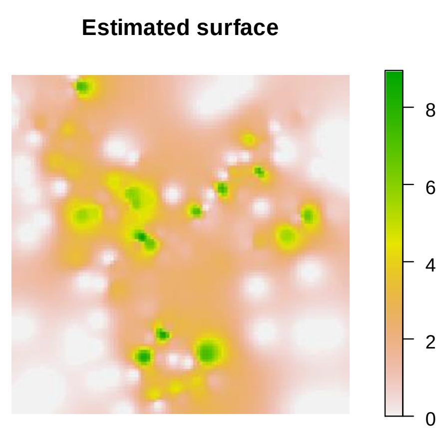
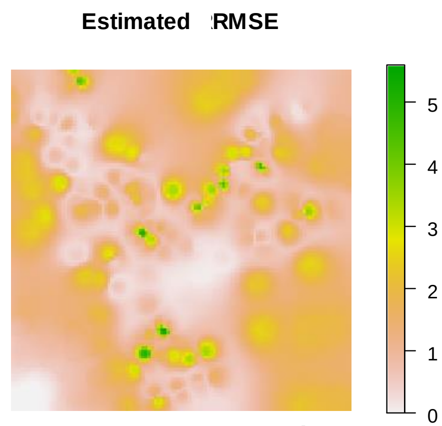




Two-phase direct interpolator





Simulation study

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Real surface

- Public forest estate in Val di Sella region
- Size 604.5 ha, trees 1213
- $y(p)$ basal area (m^2) within a plot of radius 13 m centred at p of trees taller than 1.30 m with stem diameter at breast height greater than *5*

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- Unit square
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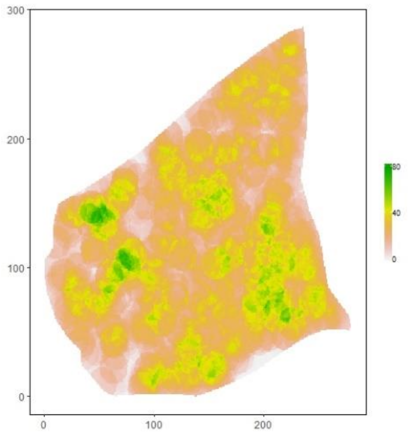

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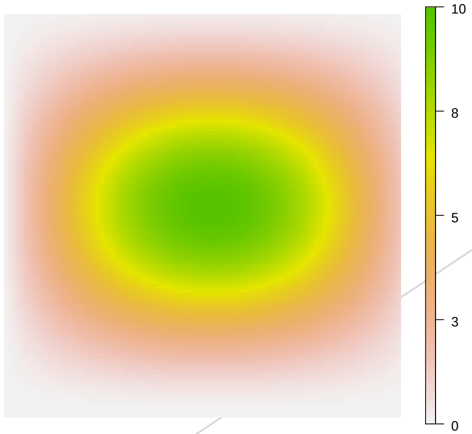

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For each first and second phase sampling effort, sampling was independently replicated 1000 times to empirically determine:

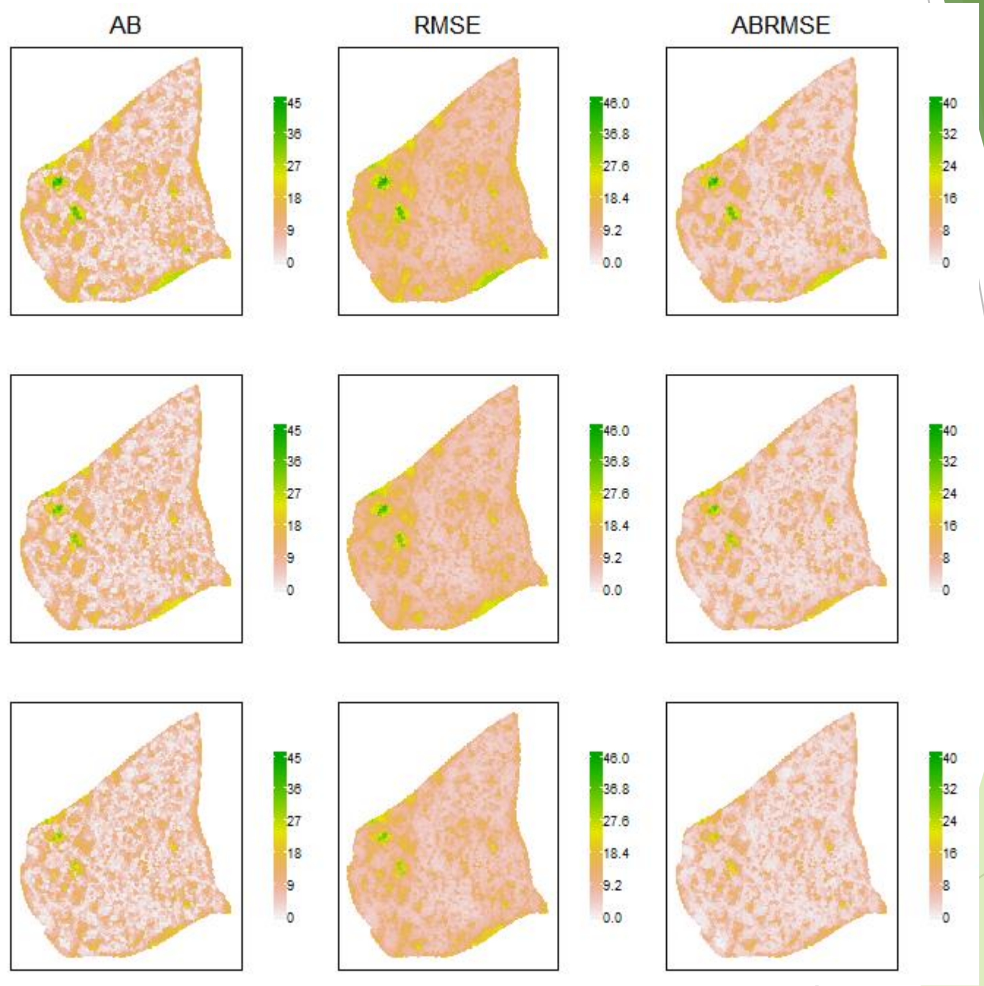
- the absolute bias (AB)
- the root mean square error (RMSE)
- the absolute bias of root mean square error (ABRMSE)

Simulation results – Real surface

first phase points: 100
second phase points: 20

first phase points: 225
second phase points: 45

first phase points: 400
second phase points: 80



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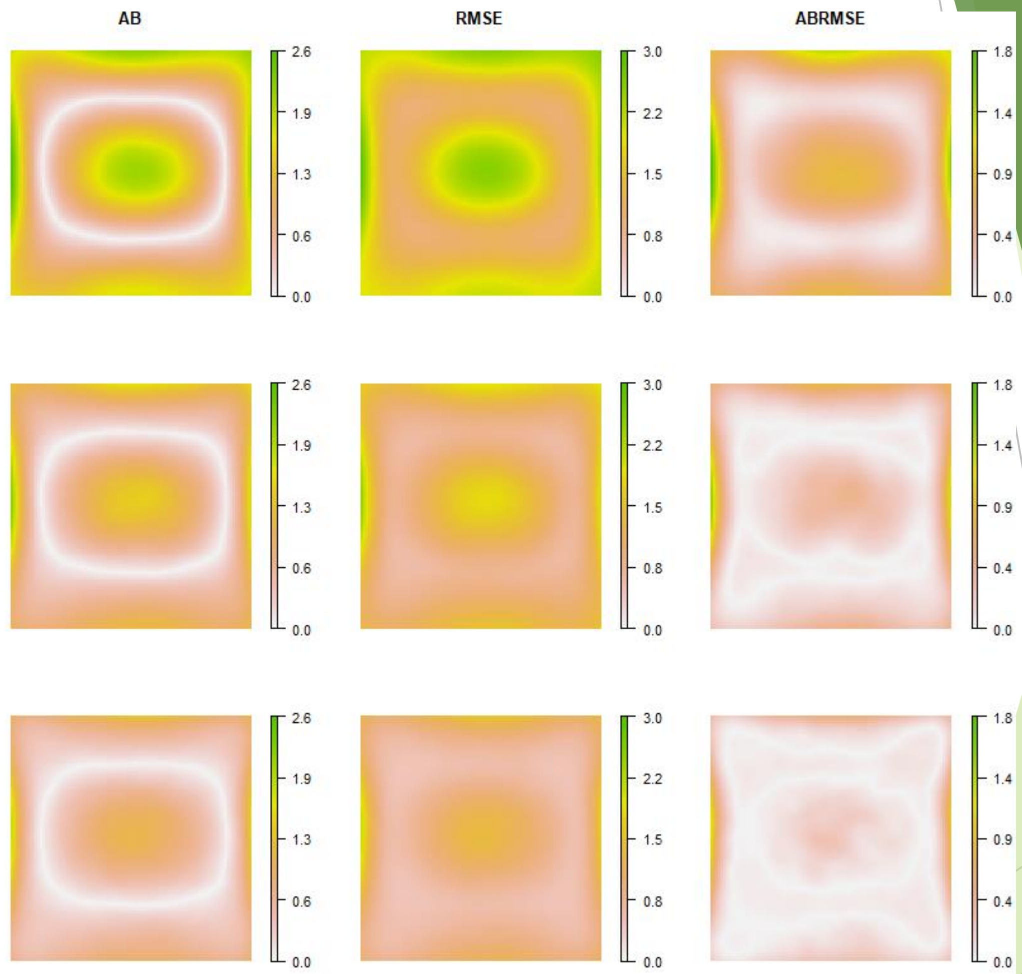

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Work in progress on simulation study…

Sampling

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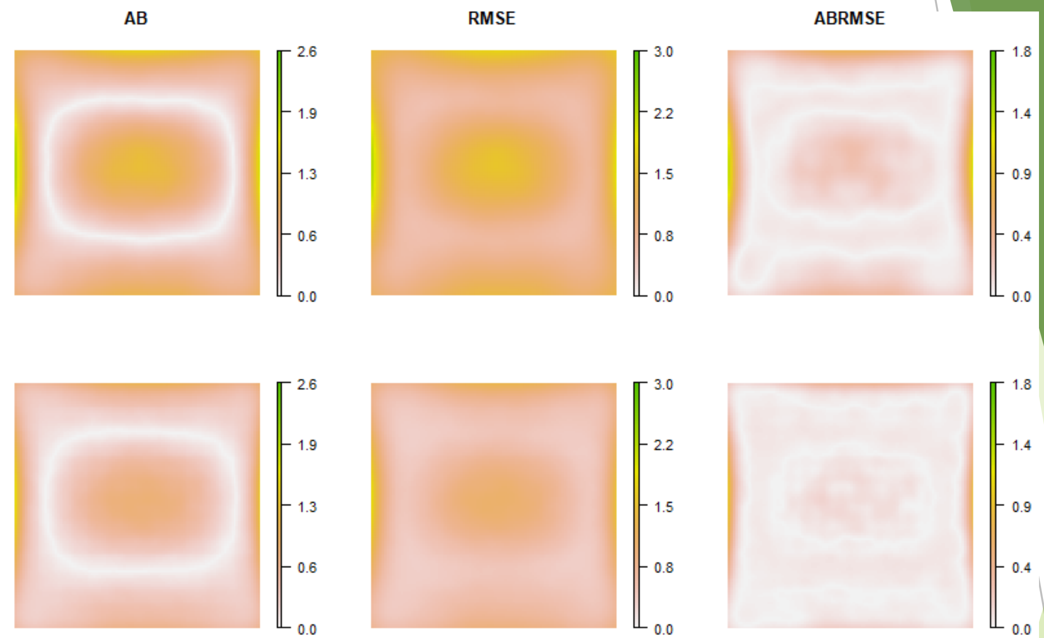
Second phase:

- simple random sampling without replacement with a sampling fraction of 50%

- One Per Stratum Sampling with sampling fraction 20% and 50%

Simulation are still running!

Simulation results – Artificial surface



first phase points: 100
second phase points: 50

first phase points: 225
second phase points: 112

first phase points: 400
second phase points: 200

Simulation are still running!

AB: absolute bias, RMSE: root mean square error
ABRMSE: absolute bias of root mean square error

Joined work with
L. Fattorini, M. Marcheselli, C. Pisani, L. Pratelli

Thanks for attention

Reference paper

L. Fattorini, M. Marcheselli, C. Pisani, L. Pratelli (2017). *Design-based asymptotics for two-phase sampling strategies in environmental surveys*, Biometrika, 104, 195-202.