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The marginal impact of auxiliary totals in calibration

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Introduction

Calibrated estimator (Deville & Särndal, 1992)
is widely used for deriving survey estimates

Main reasons (Särndal, 2007):

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- achieve consistent estimates
    - very important for NSIs
    - means for promoting credibility in published statistics
    - better provision of administrative data and registers
- just a set of sampling weights

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- ✗ problem of convergence for the constrained optimization problem
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Impact of auxiliary totals

- best fitting
- Shapley decomposition (Shapley, 1953)

Calibrated estimator

"[...] a strong correlation between the auxiliary variables and the study variable means that the weights that perform well for the auxiliary variables also should perform well for the study variables".

Deville Särndal (1992) p. 376.

$$\hat{Y}_{CAL} = \sum_{k \in s} y_k \; w_k = \sum_{k_s} y_k \; d_k \; \gamma_k$$

$$\begin{cases} \min_{w_k} \; \sum_{k \in s} G\left(w_k, d_k\right)/q_k \\ \sum_{k \in s} w_k \; \boldsymbol{x}_k = \boldsymbol{X} \end{cases}$$

$$\text{var}\left(\hat{Y}_{CAL}\right) = \sum_{k \in s} \sum_{\ell \neq k} \frac{\Delta_{k\ell}}{\pi_{k\ell}} \left(e_k w_k\right)\left(e_\ell w_\ell\right)$$

Shapley decomposition

based on the well-known concept of **Shapley value** (Φ) in cooperative game theory (Shapley, 1953)

$$\Phi\left(i,\nu\right)=\sum_{S\subseteq N}\frac{\left(|N|-|S|\right)!\left(|S|-1\right)!}{|N|!}\left(\nu\left(S\right)-\nu\left(S\right)/\left\{i\right\}\right)$$

re-assign the profits generated by a coalition of N players in proportion to the contribution that each player has made to the coalition itself.

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  - **auxiliary totals in calibration**

Example

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	model	r^2
1.	Y	0.000
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3.	$Y \sim Z$	0.122
4.	$Y \sim X$	0.503
5.	$Y \sim Z + W$	0.379
6.	$Y \sim X + W$	0.808
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Shapley value

$$\begin{array}{lcrr} \Phi\left(X, r^2\right) & = & 0.4745 & (62.8\%) \\ \Phi\left(Z, r^2\right) & = & 0.0695 & (9.2\%) \\ \Phi\left(W, r^2\right) & = & 0.2110 & (27.9\%) \\ \hline & & 0.7550 & (100.0\%) \end{array}$$

Application on calibration context

- players $\rightarrow$ auxiliary totals
- characteristic function $\nu(\cdot)$
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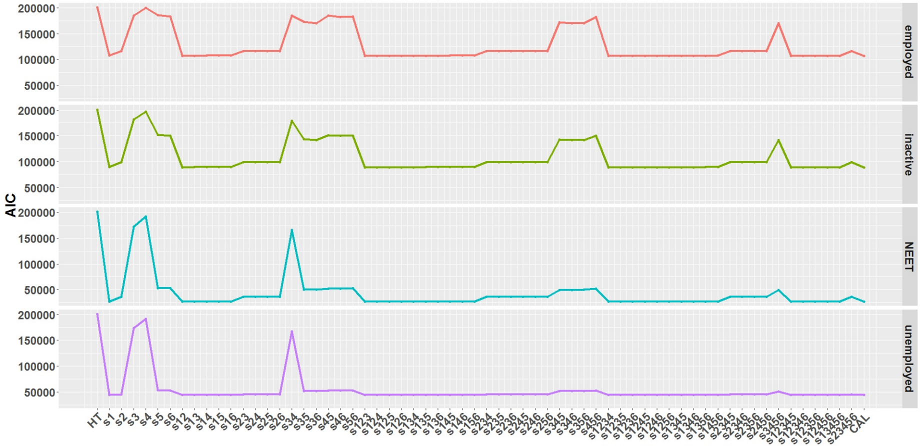
re-assign the change generated by a **calibration system of N auxiliary totals (groups of auxiliary totals)** in proportion to the contribution that each auxiliary totals (group of auxiliary totals) has made to the coalition itself in terms of **change in the estimates** and **change in variance**.

Application on Italian LFS 2013-2016

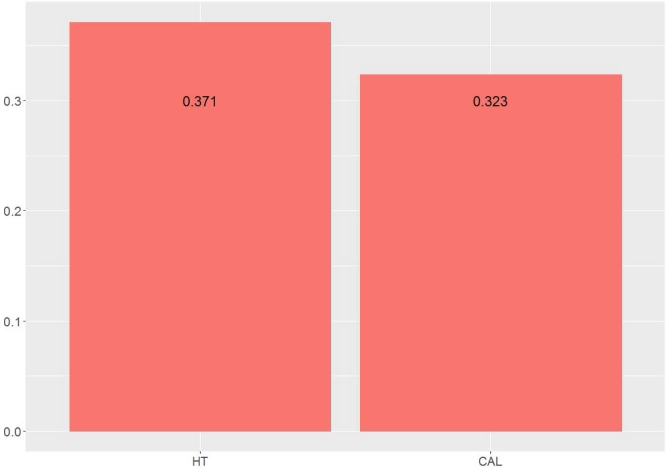
- players $\rightarrow$ auxiliary totals
  - X1 population by region, sex and 14 age classes [28x21]
  - X2 population by province, sex and 5 age classes [120x21]
  - X3 population by metropolitan city and 5 age classes [30x21]
  - X4 population of foreigners (M, F, UE, no-UE) by region [4x21]
  - X5 population by region and rotational group [4x21]
  - X6 population by region, sex and month in the quarter [6x21]
- characteristic function $\nu(\cdot)$
  - statistics (Y)
    - employment rate
    - unemployment rate
    - inactivity rate
    - NEET rate
  - domains
    - national (NUTS-0)
    - regional (NUTS-2)

Best fitting criterion



Change in variance ($cv\%$)

Employment rate - Italy, Q1-2013



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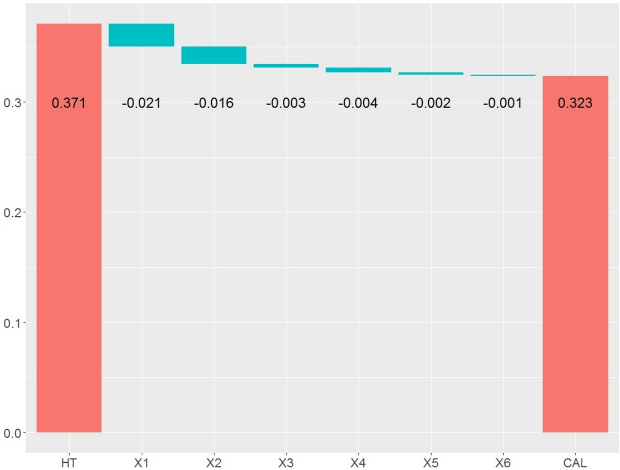

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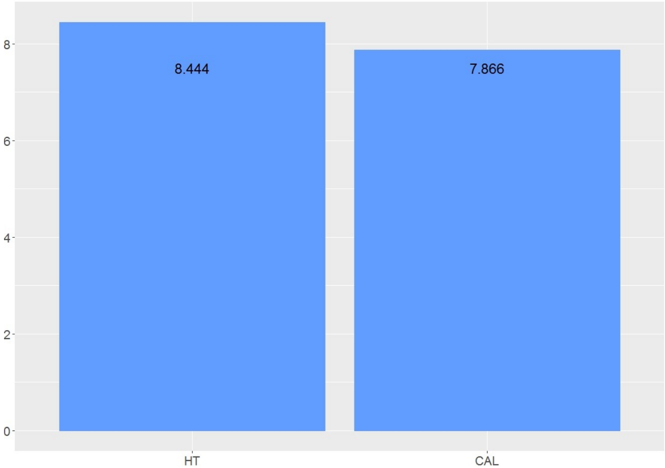

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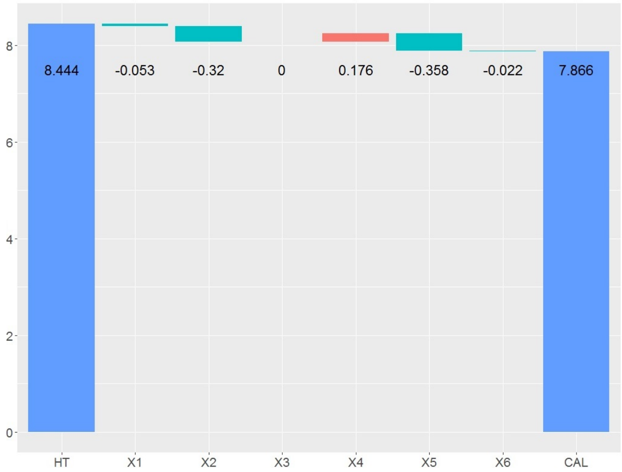
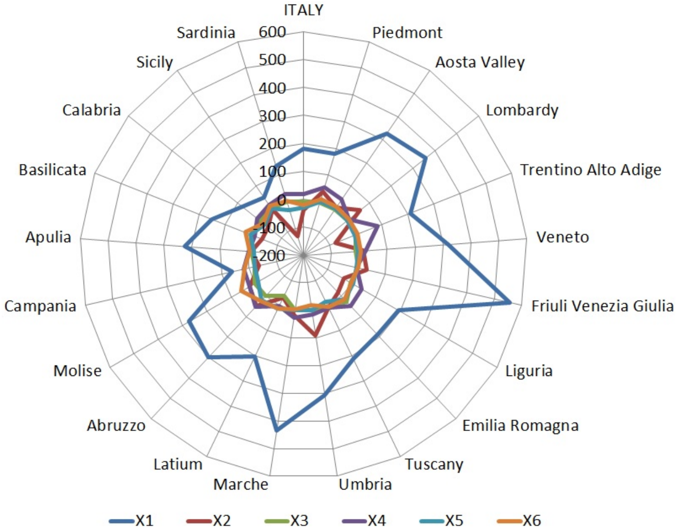
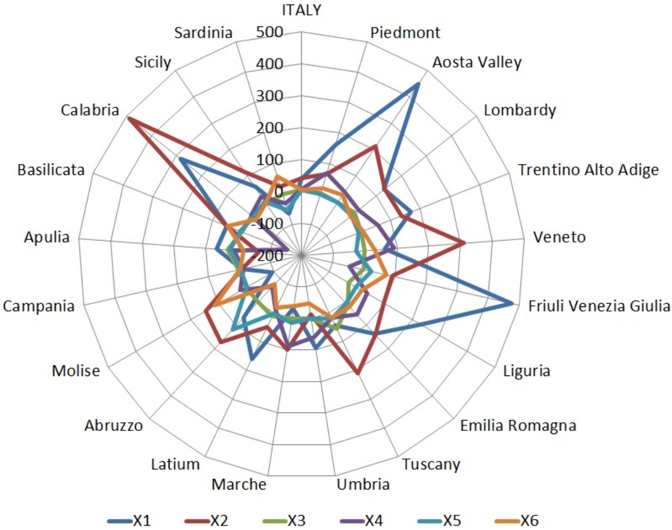


Marginal impact on estimates, Q1-2015

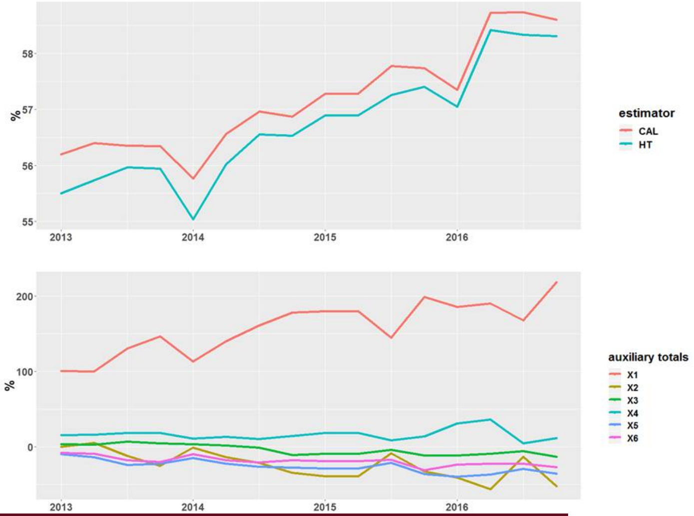


Marginal impact on variance (*cv%*), Q1-2015



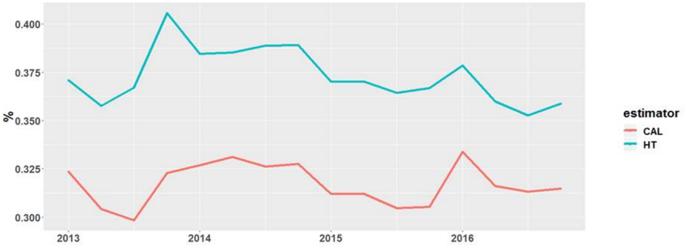
Marginal impact during the time (*estimate*)

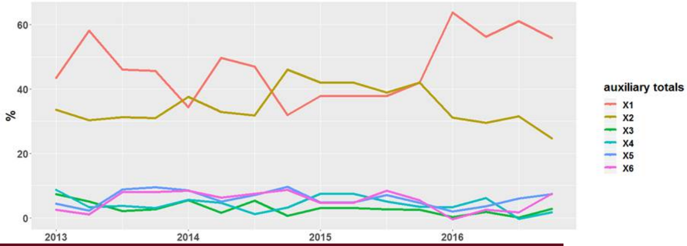
Employment rate - Italy



Marginal impact during the time ($cv\%$)

Employment rate - Italy





Conclusion

- Shapley decomposition applied to the calibration context can help to better understand the role in the estimation of auxiliary totals
- the use of auxiliary totals must be analyzed both in estimates and variance perspective
- the impact of auxiliary totals change w.r.t statistics and domains

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