# Composite Indicators for Measuring Socio-economic Phenomena

## The Performance Interval Approach

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# Contents

# Introduction

In the last years, there has been a growing interest in composite indices, whether they be social, socio-economic or environmental indices.

This attention is the result of the report by Stiglitz-Sen-Fitoussi Commission on “The Measurement of Economic Performance and Social Progress”, although the tradition of composite indices dates back to the 1970s.

As is known, a composite index is a mathematical combination (or aggregation as it is termed) of a set of individual indicators that represent the different dimensions of a phenomenon to be measured.

Therefore, composite indices aim to measure phenomena that are characterized by a multiplicity of aspects or dimensions, such as development, quality of life, or well-being.

# The performance interval approach

The performance interval approach is a new approach to the composite indices construction which consists in computing an interval of possible values, for each statistical unit, rather than a single value.

The interval is called '*performance interval*' and it is constructed depending on the level of compensability of individual indicators.

The performance interval generates a lower and upper bound for the composite index:

- one bound corresponds to the hypothesis of full-compensability of individual indicators (*full compensatory composite index*),
- the other bound corresponds to the hypothesis of non-compensability of individual indicators (*non-compensatory composite index*).

So, the centre of the interval (midpoint) may be regarded as the value of the composite index under the hypothesis of partial compensability (*partially compensatory composite index*).

# The performance interval approach

Note that the length of the interval can be considered as a measure of imbalance of individual indicators.

If individual indicators are perfectly balanced, the performance interval reduces to a single point, and the value of the composite index is independent of any hypothesis on the level of compensability of individual indicators.

The greater the length of the interval, the greater the imbalance of individual indicators and the larger the difference between the value of the full-compensatory composite index and the value of the non-compensatory composite index.

# The performance interval approach

**Special cases of the power mean of order *r***

| Order | Formula | Aggregation function | Approach | Penalization | |
|---|---|---|---|---|---|
| | | | | Intensity | Direction |
| $r\to-\infty$ | $M_i^{-\infty}=\min_j(y_{ij})$ | Minimum | Non-compensatory | Maximum | Downward |
| $r$=-1 | $M_i^{-1}=\left(\sum_{j=1}^{m}\frac{w_j}{y_{ij}}\right)^{-1}$ | Harmonic mean | Partially compensatory | High | Downward |
| $r\to0$ | $M_i^{0}=\prod_{j=1}^{m}y_{ij}^{w_j}$ | Geometric mean | Partially compensatory | low | Downward |
| $r$=1 | $M_i^{1}=\sum_{j=1}^{m}y_{ij}w_j$ | Arithmetic mean | Compensatory | None | - |
| $r$=2 | $M_i^{2}=\left(\sum_{j=1}^{m}y_{ij}^2w_j\right)^{\frac{1}{2}}$ | Quadratic mean | Partially compensatory | low | Upward |
| $r$=3 | $M_i^{3}=\left(\sum_{j=1}^{m}y_{ij}^3w_j\right)^{\frac{1}{3}}$ | Cubic mean | Partially compensatory | High | Upward |
| $r\to+\infty$ | $M_i^{+\infty}=\max_j(y_{ij})$ | Maximum | Non-compensatory | Maximum | Upward |

$$M_i^{-\infty}\le\ldots\le M_i^{-1}\le M_i^{0}\le M_i^{1}\le M_i^{2}\le M_i^{3}\le\ldots\le M_i^{+\infty}$$

# The performance interval approach

**How to calculate the performance interval**

1) 'positive' composite index (e.g., development index)

Let **Y**={$y_{ij}$} be the normalized matrix with *n* rows (statistical units) and *m* columns (normalized indicators).

For each unit *i* (*i*=1, …, *n*), we have:

Upper bound - Full-compensatory composite index

$$UB_i = \frac{\sum_j^m y_{ij}}{m}$$

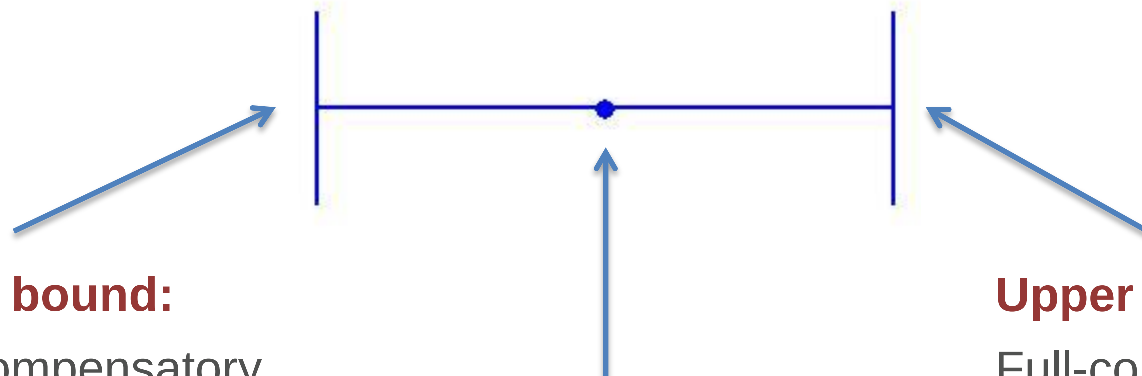
Lower bound - Non-compensatory composite index

$$LB_i = \min_j\{y_{ij}\}$$

# The performance interval approach

**How to calculate the performance interval**

1) ‘positive’ composite index (e.g., development index)



**Lower bound:**

Non-compensatory composite index

**Midpoint:**

Partially compensatory composite index

**Upper bound:**

Full-compensatory composite index



Istat

# The performance interval approach

**How to calculate the performance interval**

2) ‘negative’ composite index (e.g., poverty index)

Let **Y**={$y_{ij}$} be the normalized matrix with *n* rows (statistical units) and *m* columns (normalized indicators).

For each unit *i* (*i*=1, …, *n*), we have:

Upper bound - Non-compensatory composite index

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# The performance interval approach

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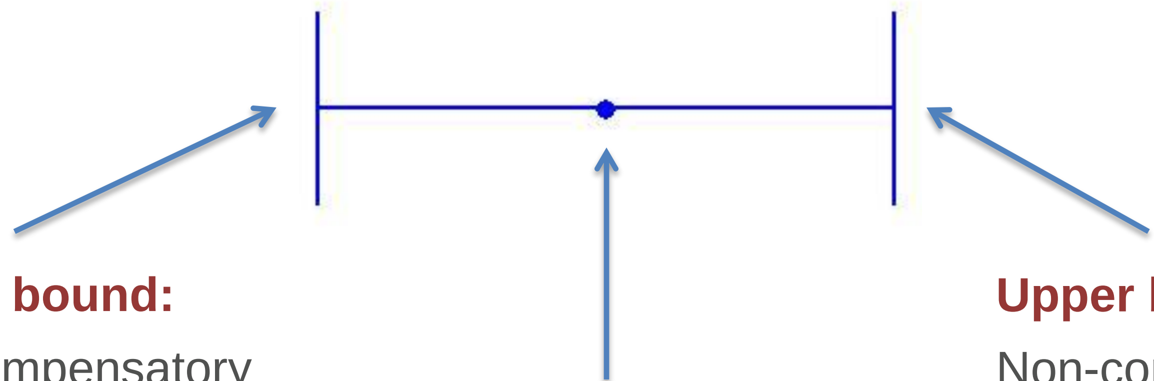


**Lower bound:**

Full-compensatory composite index

**Midpoint:**

Partially compensatory composite index

**Upper bound:**

Non-compensatory composite index

# An application to well-being data

**Individual indicators of well-being in the Italian regions - Year 2017 (Source: Istat, 2018)**

| Regions | Income per capita (euro) | Life expectation (years) | Graduates aged 30–34 (%) | Unemployment rate (%) | Separate waste collection (%) |
|---|---|---|---|---|---|
| Piemonte | 20.727 | 82,5 | 26,4 | 9,1 | 59,3 |
| Valle d'Aosta | 20.901 | 82,0 | 25,2 | 7,8 | 61,1 |
| Liguria | 21.639 | 82,7 | 23,7 | 9,5 | 48,8 |
| Lombardia | 22.419 | 83,3 | 33,7 | 6,4 | 69,6 |
| Trentino Alto Adige | 23.193 | 83,8 | 29,1 | 4,4 | 72,0 |
| Veneto | 20.350 | 83,4 | 27,6 | 6,3 | 73,6 |
| Friuli Venezia Giulia | 20.562 | 83,0 | 28,7 | 6,7 | 65,5 |
| Emilia Romagna | 22.463 | 83,2 | 29,9 | 6,6 | 63,8 |
| Toscana | 20.275 | 83,3 | 28,3 | 8,6 | 53,9 |
| Umbria | 18.038 | 83,3 | 29,7 | 10,6 | 61,7 |
| Marche | 18.722 | 83,3 | 33,0 | 10,6 | 63,2 |
| Lazio | 19.366 | 82,5 | 30,1 | 10,7 | 45,5 |
| Abruzzo | 16.284 | 82,6 | 25,8 | 11,7 | 56,0 |
| Molise | 14.416 | 82,3 | 26,1 | 14,6 | 30,7 |
| Campania | 13.153 | 81,1 | 21,4 | 20,9 | 52,8 |
| Puglia | 13.932 | 82,7 | 22,2 | 18,9 | 40,4 |
| Basilicata | 13.483 | 82,3 | 29,2 | 12,8 | 45,3 |
| Calabria | 12.656 | 82,1 | 20,7 | 21,6 | 39,7 |
| Sicilia | 13.286 | 81,6 | 19,1 | 21,5 | 21,7 |
| Sardegna | 15.240 | 82,8 | 23,6 | 17,0 | 63,1 |

# An application to well-being data

**Normalized indicators of well-being in the Italian regions (μ=100; σ=10) - Year 2017**

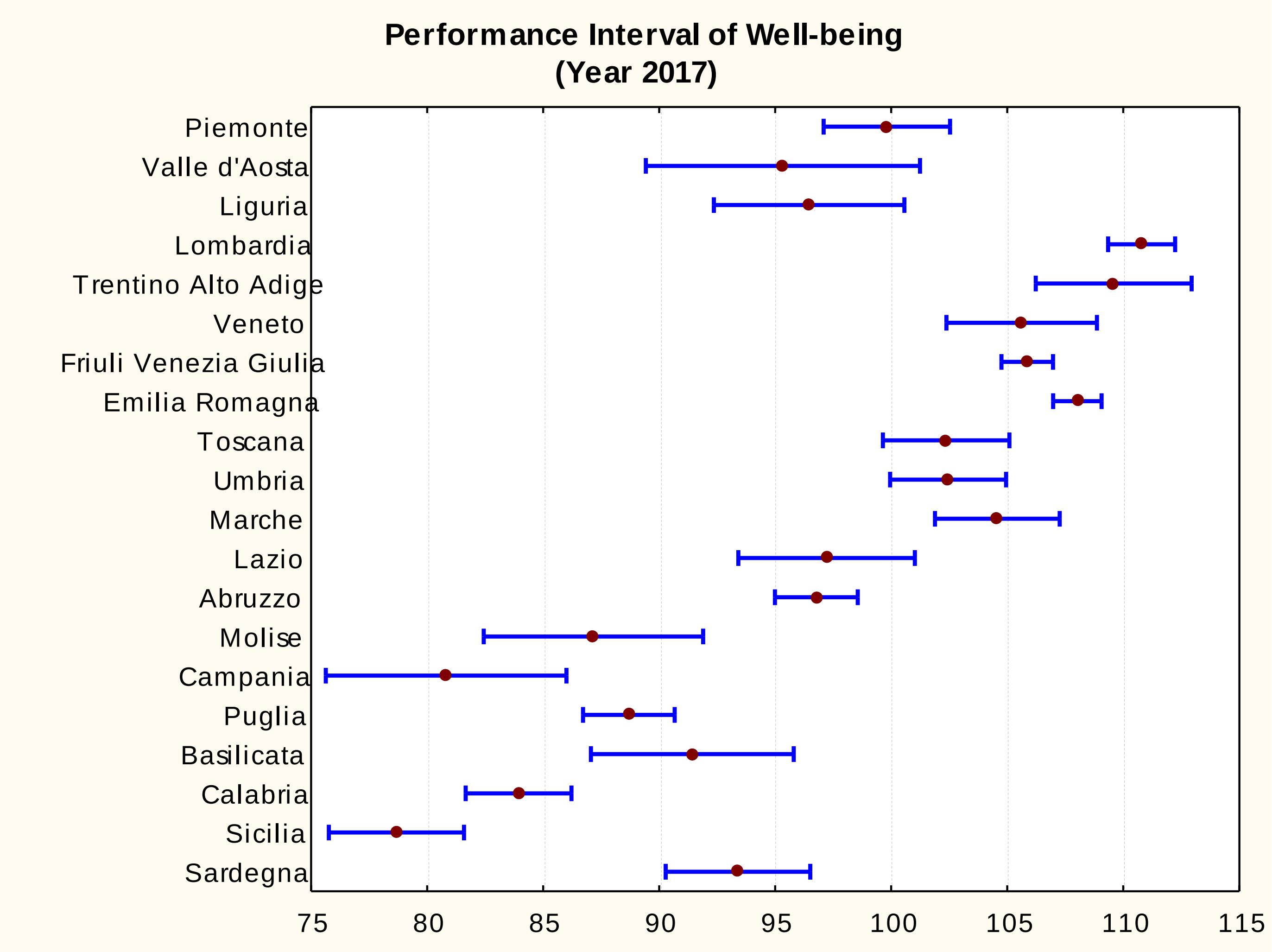
| Regions | Income per capita (euro) | Life expectation (years) | Graduates aged 30–34 (%) | Unemployment rate (%) | Separate waste collection (%) |
|---|---|---|---|---|---|
| Piemonte | 107,6 | 97,1 | 99,3 | 105,1 | 103,6 |
| Valle d'Aosta | 108,1 | 89,4 | 96,2 | 107,5 | 105,0 |
| Liguria | 110,1 | 100,2 | 92,4 | 104,3 | 95,9 |
| Lombardia | 112,3 | 109,3 | 118,0 | 110,1 | 111,3 |
| Trentino Alto Adige | 114,5 | 117,0 | 106,2 | 113,9 | 113,1 |
| Veneto | 106,5 | 110,9 | 102,4 | 110,3 | 114,2 |
| Friuli Venezia Giulia | 107,1 | 104,8 | 105,2 | 109,6 | 108,2 |
| Emilia Romagna | 112,5 | 107,8 | 108,3 | 109,8 | 107,0 |
| Toscana | 106,3 | 109,3 | 104,2 | 106,0 | 99,6 |
| Umbria | 100,0 | 109,3 | 107,8 | 102,3 | 105,4 |
| Marche | 101,9 | 109,3 | 116,2 | 102,3 | 106,5 |
| Lazio | 103,7 | 97,1 | 108,8 | 102,1 | 93,4 |
| Abruzzo | 95,0 | 98,6 | 97,8 | 100,2 | 101,2 |
| Molise | 89,7 | 94,0 | 98,5 | 94,8 | 82,4 |
| Campania | 86,1 | 75,6 | 86,4 | 83,0 | 98,8 |
| Puglia | 88,3 | 100,2 | 88,5 | 86,7 | 89,6 |
| Basilicata | 87,1 | 94,0 | 106,5 | 98,2 | 93,3 |
| Calabria | 84,7 | 91,0 | 84,7 | 81,7 | 89,1 |
| Sicilia | 86,5 | 83,3 | 80,5 | 81,8 | 75,8 |
| Sardegna | 92,0 | 101,7 | 92,1 | 90,3 | 106,5 |

# An application to well-being data

**Composite Indices of well-being in the Italian regions - Year 2017**

| Regions | Performance Interval | | Composite index | | Rank | | |
|---|---|---|---|---|---|---|---|
| | LB | UB | Midpoint | Geometric Mean | Midpoint | Geometric Mean | Midpoint-Geometric mean |
| Piemonte | 97,1 | 102,5 | 99,8 | 102,5 | 9,0 | 9,0 | 0 |
| Valle d'Aosta | 89,4 | 101,2 | 95,3 | 101,0 | 13,0 | 10,0 | 3 |
| Liguria | 92,4 | 100,6 | 96,5 | 100,4 | 12,0 | 12,0 | 0 |
| Lombardia | 109,3 | 112,2 | 110,8 | 112,2 | 1,0 | 2,0 | -1 |
| Trentino Alto Adige | 106,2 | 112,9 | 109,6 | 112,9 | 2,0 | 1,0 | 1 |
| Veneto | 102,4 | 108,9 | 105,6 | 108,8 | 5,0 | 4,0 | 1 |
| Friuli Venezia Giulia | 104,8 | 107,0 | 105,9 | 107,0 | 4,0 | 6,0 | -2 |
| Emilia Romagna | 107,0 | 109,1 | 108,0 | 109,0 | 3,0 | 3,0 | 0 |
| Toscana | 99,6 | 105,1 | 102,4 | 105,0 | 8,0 | 7,0 | 1 |
| Umbria | 100,0 | 105,0 | 102,5 | 104,9 | 7,0 | 8,0 | -1 |
| Marche | 101,9 | 107,3 | 104,6 | 107,1 | 6,0 | 5,0 | 1 |
| Lazio | 93,4 | 101,0 | 97,2 | 100,9 | 10,0 | 11,0 | -1 |
| Abruzzo | 95,0 | 98,6 | 96,8 | 98,5 | 11,0 | 13,0 | -2 |
| Molise | 82,4 | 91,9 | 87,2 | 91,7 | 17,0 | 16,0 | 1 |
| Campania | 75,6 | 86,0 | 80,8 | 85,7 | 19,0 | 19,0 | 0 |
| Puglia | 86,7 | 90,7 | 88,7 | 90,5 | 16,0 | 17,0 | -1 |
| Basilicata | 87,1 | 95,8 | 91,4 | 95,6 | 15,0 | 15,0 | 0 |
| Calabria | 81,7 | 86,2 | 83,9 | 86,2 | 18,0 | 18,0 | 0 |
| Sicilia | 75,8 | 81,6 | 78,7 | 81,5 | 20,0 | 20,0 | 0 |
| Sardegna | 90,3 | 96,5 | 93,4 | 96,3 | 14,0 | 14,0 | 0 |
| **Mean absolute difference** | | | | | | | **0,80** |
| **Spearman's rank correlation** | | | | | | | **0,980** |

# An application to well-being data



# Conclusions

In this work, we proposed a new approach to the composite indices construction which consists in computing a ‘performance interval’ for each statistical unit, rather than a single value.

The interval is constructed depending on the level of compensability of individual indicators and the midpoint may be regarded as the value of the composite index under the hypothesis of partial compensability (partially compensatory composite index).

The performance interval approach is independent of the normalization method, and it allows to release the assumption on the level of compensability (or substituitability) of individual indicators.

The innovative aspect of the method is that it can be used both for calculating a composite index and for constructing a value range, i.e. an interval of possible values.

# References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